Theoretical Analysis on the Globalization of R&D With application on Japanese Firms[1]

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1. Introduction

Globalization of research and development (R&D) refers to a firm locating research and development facilities outside of its home country. Firms trading internationally will usually develop their products in their home country. Over the past decade, this is no longer the only option, we observe many Multi-National Corporations (MNCs) taking part in R&D globalization. In a research paper by Kuemmerle (1999), it was noted that from 1965 to 1995, the amount of international (globalized) R&D conducted by the sample of 32 international firms has grown by 400%.

As firms become globalized, competition is taken to the international level research and development has to be done at the global level for products to fit the world market. As geographical regions become irrelevant in the world, firms may seek out new R&D locations where local talents are of international quality and may be relatively cheaper. R&D conducted outside of the home country may also inject new ideas and provide the firm with innovative products suitable for the global market. Researchers from developing countries may be able to produce better products for the developing world. This is important for MNCs as developing countries such as India, China and Brazil represents a large part of the world market.

One prominent example of a MNC which has partaken in much global R&D is General Electric whose headquarters are located in Fairfield, Connecticut, USA. Its primary research facility is located in Niskayuna, New York. General Electric (GE)'s research and development division, GE Global Research maintains four significant laboratories in Bangalore, Shanghai, Munich and Rio de Janeiro. The John F. Welch Technology Centre in Bangalore is GE's first laboratory outside of the USA, it was founded in September 2000 and currently houses over 4000 engineers and scientists. The newest research center in Rio de Janeiro is part of a $500 million USD investment made by GE in Brazil and is scheduled to start operation in 2011. This center will house 200 engineers and researchers when fully operational.

1.1. Types of overseas R&D

The objective of R&D conducted overseas can be different across firms; we can largely classify overseas R&D into two main types. The first type is used to modify products to the local consumer market and production process This type of R&D is termed differently in several papers, in Ito and Wakasugi (2007) it is known as "Support Oriented R&D", in Kuemmerle (1999) it is "Home-base Exploiting R&D" and in Todo and Shimizutani (2008) the term used was "Adaptive R&D". This form of R&D adds value to the product in the local market, and is tactical in nature; it is conducted so as to create more value for an existing product in the local market. This type of R&D does not result in innovation and will not create value for the firm outside of the market where the localization is intended for.

The second type of R&D involves the development of new products using the host country's expertise. Similarly, it is know by different names. In Ito and Wakasugi (2007) it was known as "Knowledge Sourcing R&D", Kuemmerle (1999) terms it as "Home-base Augmenting R&D" and Todo and Shimizutani (2008) refers to it as "Innovative R&D". This form of R&D is strategic in nature; it has the potential to develop products which can be sold to the global market. The value created by this type of R&D is not restricted to the market it is developed in. For simplicity, we will use the terms coined by Todo and Shimizutani (2008) in this paper. From here on, we will refer to the first type of R&D as "Adaptive" and the second type as "Innovative". Several researchers have broached this subject from an empirical point of view. In the next section, we explore three empirical works which analyzed foreign R&D between different countries and firms.

1.1.1. Empirical research related to foreign R&D

Kuemmerle (1999)'s paper titled: "Foreign direct investment in industrial research in the pharmaceutical and electronics industries - results from a survey of multinational firms", surveyed 32 large multinational electronics and pharmaceutical firms from 1965 to 1995 (including 12 Japanese firms). He distinguishes between two types of R&D, Home Base Augmenting (HBA) and Home Base Exploiting (HBE). HBA is similar to innovative R&D and HBE is like adaptive R&D. Kuemmerle found that 38% of labs (at 1995) were of the HBA type. Over the 30 year period, Japanese labs based in the United States increased from 0 to 33, from 0 to 18 in Europe and from 0 to 7 in the rest of the world. Japanese firms are starting to invest in R&D abroad over the 30 year period. We can observe from his paper that the amount of overseas investment is increasing over his survey period and more firms are investing in HBA. Unfortunately, the ratio of HBA to HBE in each country was not provided in Kuemmerle's paper.

Todo and Shimizutani (2009)'s "R&D intensity for innovative and adaptive purposes in overseas subsidiaries: Evidence from Japanese multinational enterprises" used firm level panel data for Japanese multinational firms. In their paper, R&D is classified into "Innovative R&D" (basic and applied research) which exploits local knowledge and "Adaptive R&D" (development and design). Some factors which Todo and Shimizutani found was significant to the amount of overseas R&D are:

1) Size of host country was found to positively affect both types of overseas R&D activities. A larger country tends to engage in more overseas R&D.

2) Geographic distance from the home country has a negative impact. The home country is less likely to engage in R&D in countries which are further away.

3) Parent firm's knowledge positively affects only adaptive R&D. A firm with a larger knowledge base in their own home country is more likely to conduct the adaptive type of R&D in a foreign country. We can also interpret this as a possibility that they will choose adaptive over innovative if they already have a large knowledge base at home.

Finally in Song, Asakawa and Chu (2011)'s "What determines knowledge sourcing from host locations of overseas R&D operations?: A study of global R&D activities of Japanese multinationals", the authors looked at the determinants of knowledge sourcing from host locations of overseas operations. They investigate the extent where a foreign subsidiary may communicate and seek guidance from the parent company. 2 main factors were considered. The first factor is "Technology capabilities", an inverted U relationship was found, labs with medium technology capabilities were most likely to source knowledge at the host location. The second factor is "Embededness in the local scientific community"; this factor was found to have a positive relationship with the amount of knowledge sourcing.

The three papers discussed above illustrate that there is an interest in the empirical literature on the issue of overseas R&D and its role. However, there is a lack of theoretical analysis on overseas R&D (innovative R&D in particular) and its effect on the host and parent country. In this report, we will develop a theoretical model to address this issue. In the next section, we start by considering the benefits of overseas R&D in the next section.

1.2. The benefits of innovative R&D

As noted in the literature above, the amount of innovative R&D has increased over the years. In Kuemmerle (1999), it was found that 38% of overseas labs were of this type. It is widely agreed upon that innovative R&D would be beneficial to the host country. Such R&D labs would result in more employment and spillovers into the local scientific community. On the other hand, an international firm can also reap benefits from engaging in R&D in a foreign country. The development of new products in these markets often can open up new product segments which find its use in other parts of the world. Often local researchers can come up with new innovations which a researcher in a developed country may not have thought of. We categorize these advantages into 3 key types:

1) It adds value to the product locally and improves the firm's competitiveness in the local market. If it is conducted in a large emerging developing country like China, the firm would be able to exploit a large market of consumers eager for high-tech products catered to their needs.

2) By employing local talent, it can dampen the R&D competition from local firms. An MNC would have the relative advantage when employing local talent as they are more prestigious firms to work for and can pay better wages than local firms. In developing countries, MNC can potentially act as the first mover in the local market and employ the best local talents. This will significantly reduce the talent pool available for local firms and reduce their ability to catch up in terms of R&D.

3) The locally developed product may have potential for global use. Local researchers may be able to think outside the constraints of the home office. This can result in spillovers to other international markets including the firm's home market. One successful example of this is General Electric; we will discuss one of GE's successful innovations in the next section. We will also look at the case of the Tato Nano project in India.

1.2.1. GE: Ultrasound devices

One of the key businesses of General Electric is in healthcare equipment. GE Healthcare's history dates back from the 20th century; it is currently worth $17 billion USD and employs over 46,000 people worldwide. One of GE Healthcare's product range are medical imaging devices such as ultrasound machines. In the 1990s, GE sold conventional ultrasound devices. Each device costs more than $100,000 USD in China. These were mostly purchased by sophisticated hospitals in the large cities of China; regional hospitals could not afford such equipment. In 2002, a cheap portable ultrasound device was developed by GE's research team in China; its price was $30,000 to $40,000 USD. The new portable device was sold to rural clinics in China and even to ambulance squads and emergency rooms in the US. The product has found a market outside of China and is sold worldwide. In 2007, a dramatically cheaper model priced at only $15,000 USD was launched.

Figure 1: GE's portable ultrasound device

The Chinese research team recognized that the Chinese market required a cheaper device which could easily transported to bring medical services to rural areas. The local team was in tune with the needs of the local market and thus was able to produce a product more suited for local use. Although the new device did not produce images which were better than conventional ultrasound machines, it was more affordable. Products need to meet specific needs and budgets of their buyers. Consumers in developing countries are happy with "high-tech solutions that deliver decent performance at an ultralow cost - a 50% solution at 15% price".

1.2.2. The Tata Nano Project in India

The idea to develop the cheapest car in world was initiated by Tato Motors (an Indian based automobile manufacturing company) in 2003. The creation of Tato Nano gathered many technology leaders in the motoring industry from all over the world. The Tato Nano was released in 2009 and is current retailing for a price between US$3000 to US$4000 in India. Many German companies (such as Bosch) were involved in the development of parts.

Figure 2: The Tato Nano

In order to achieve the low price, the reengineering of high-end parts with "Good Enough" parts was required. Being involved in the project has greatly enhanced Bosch technical know-how for small car production. "Nano's extreme cost target needed a radical new approach. For Bosch, the downscaling experience of Nano has opened up new business opportunities.", global chairman of Bosch's automotive group Bernd Bohr. By engaging in developing products for the developing market, Bosch’s was able to rejuvenate its research and create new product areas and niches.

1.3. Japanese firms and their outward (foreign) R&D

Since there are many benefits to be reaped from foreign or outward R&D, we see many MNCs from the United States and Europe setting up foreign R&D subsidiaries in emerging markets such as China and India. Although many firms have taken part in global R&D, we observe that firms originating from different countries may not invest the similar amounts. The table below shows the amount of outward R&D (conducted outside of the host country) as a percentage of the national total for 5 OECD countries:

Outward R&D as a percentage of the national total - OECD

	2002	2003	2004	2005	2006	2007	2008
Germany	..	23.7	..	29.9	..	24.5	..
Italy	3	2.8	..	..	..	..	..
Japan	3.1	2.9	2.9	2.7	2.9	2.8	..
Switzerland	..	..	99.4	..	..	..	131.6
United States	10.9	11.4	12.4	12.2	11.9	12.8	13

Data Taken From: OECD Statistics on Measuring Globalisation, Last updated March 2011

Figure 3: Outward R&D of some OECD countries

We find that Japanese firms are slow to join this trend. Although we observe that Japan has consistently invested in outward R&D over the 6 year period, the amount invested represents a small percentage of Home-R&D when compared with countries like Germany and the United States. The table below shows the amount spent by Japanese firms in other countries on R&D. A large concentration of this is in the West and more developed countries.

Amount Spent (in Yen) on outward R&D in each Country/Region

	2002	2003	2004	2005	2006	2007
World	353 816	337 447	347 870	349 949	387 301	387 963
United States	208 652	165 607	148 426	161 812	195 058	195 515
European Union (15)	103 440	121 636	97 519	115 537	114 212	105 247
France	8 929	..	9 272	8 666	9 217	7 300
Germany	16 837	20 883	14 270	14 033	21 665	22 283
United Kingdom	33 773	35 192	37 514	29 635	26 106	29 972
Non-OECD Asia	29 997	36 818	73 188	49 780	51 445	62 405
Western and Eastern Europe	707	1 178	8 497	5 332	5 084	2 577

Data Taken From: OECD Statistics on Measuring Globalisation, Last updated March 2011

Figure 4: Spending in outward R&D by Japan

Although Japan's overseas R&D ratio increased from 2.9% in 1997 to 4.1% in 2002, the share of foreign affiliates in industrial R&D remains at less than 5%, Song, Asakawa and Chu (2011). In the next table, we can compare the amount of Japanese R&D conducted overseas to local R&D.

HOME AND OVERSEAS R&D BY INDUSTRY AND BY YEAR

	Aggregate R&D intensity (%)			
	Home R&D	Overseas R&D	Innovative overseas R&D	Adaptive overseas R&D
Year				
1996	18.47	0.92	0.51	0.32
1997	18.41	1.13	0.60	0.30
1998	19.86	1.44	0.56	0.45
1999	18.04	1.39	0.62	0.48
2000	16.88	1.36	0.66	0.49
2001	19.46	1.48	0.72	0.59
Total	18.46	1.30	0.61	0.44

Notes: This table presents the aggregate R&D intensity by industry, by year, and in total. The aggregate intensity of home and overseas R&D is defined as the ratio of the aggregate R&D expenditure of parent firms and overseas subsidiaries, respectively, to the aggregate value added of parent firms in percentages. Correspondingly, the aggregate intensity of overseas innovative and adaptive R&D is the ratio of aggregate expenditure on overseas innovative and adaptive R&D, respectively, to parent firms' value added. These numbers above are based on 2,671 firm-year observations for Japanese MNE's used in our regression.

Taken from Todo and Shimizutani (2008)

Figure 5: Japanese R&D by type

We see that overseas R&D has not increased much over the years in terms of intensity. This data is further divided into innovative and adaptive R&D, we can observe that the amount of each type of R&D do not differ much. Although, Japan does engage in overseas R&D, this is relatively small compared to R&D conducted within Japan. It is also relatively small when compared to other developed countries.

The lack of Japanese overseas R&D when compared with the rest of the world raises an interesting question. What are the reasons why this is the case? We would try to answer this using economic theory.

1.4. Aims and Scope

There are three main aims of this paper:

1) Using a microeconomic and industrial organizational perspective, adapt a theoretical model to analyze the firm's strategic decision to conduct overseas R&D.

2) Analyze the Japanese situation using the results derived from our model and try to understand why they have been slow to join the localization trend.

3) Relate these results to real life and discuss some policy implications. R&D from other countries are useful for a country's development, we will discuss what a country can do to attract R&D from overseas.

The setup of the report will be as follows: we start by developing the theoretical model by introducing the differentiated Bertrand model and modifying it for our use. After deriving results from the model, we analyze these results and discuss their real life implications. This is followed by concluding remarks.

2. The Model

In this section, we present a theoretical model of competition to help us understand the motives of firm's decision to set up foreign R&D subsidiaries. The main focus of our analysis is the degree where a product is localized via R&D or the localization choice. This is made a continuous choice using the localization factor (γ) in the model. The magnitude of γ can be used as a measure of whether adaptive or innovative R&D has been undertaken. As γ gets larger; products are more localized and tailored to local tastes. If γ is zero, there is no localization at all. If γ=1, then there is full localization and we treat this as the equivalent of innovative R&D. The foreign firm will choose γ strategically (by considering the actions of its rival). Given a set of fixed existing conditions, the model can be used to derive the best strategy to be taken in terms of γ. We can also use the model to examine how to change the conditions in order to make the business environment more conducive to setting up foreign R&D subsidiaries. As a baseline for our analysis, we first introduce the Bertrand model of product differentiation price competition.

2.1. The Bertrand model of differentiated price competition

The Bertrand model of product differentiation is used widely in economics to analyse firm behaviour and strategies. For example, Eaton and Grossman (1986) used the model to discuss international trade and Symeonidis (2003) used it to analyze the effect on the amount of R&D undertaken by competitive firms. The model can be used to describe strategic competition, this fits the idea that firms compete with local firms when they enter a foreign market. The element of localization (γ) can be introduced easily in this model. The Bertrand model is a model of strategic complements; this means that increasing the amount of localization improves profits for both the local and foreign firm as it will lessen price competition between the firms. When a firm increases its own prices, it will get a decrease in demand. Conversely, if its rival firm increase prices, demand will increase. Pricing higher than rival firms does not result in a large drop in customer base, this is especially so when the amount of product differentiation is high.

In the Bertrand model, each firm strategically considers their rival's actions when making their decisions. They develop a rule called the reaction function, which is the optimal price to choose given the rival's price. In Bertrand competition, the reaction functions are upward sloping.

Take for example the linear differentiated demand functions:

$q_1 = A - \beta p_1 + \beta p_2$
$q_2 = A - \beta p_2 + \beta p_1$ (1)

Both firms experience some amount of product differentiation, its own price will decrease its demand but its rival's price will increase demand. Each firm maximizes their profit given that the rival's price is given. Profits can be written as:

$\Pi_1 = (p_1 - c)(A - \beta p_1 + \beta p_2)$
$\Pi_2 = (p_2 - c)(A - \beta p_2 + \beta p_1)$ (2)

Maximizing for profits, we can derive the reaction functions R_1 and R_2:

$R_1(q_2) \Rightarrow q_1 = (A + \beta p_2)/(2\beta)$
$R_2(q_1) \Rightarrow q_2 = (A + \beta p_1)/(2\beta)$ (3)

They each represent a firm's optimum production quantity. The reaction functions can be plotted in the following manner:

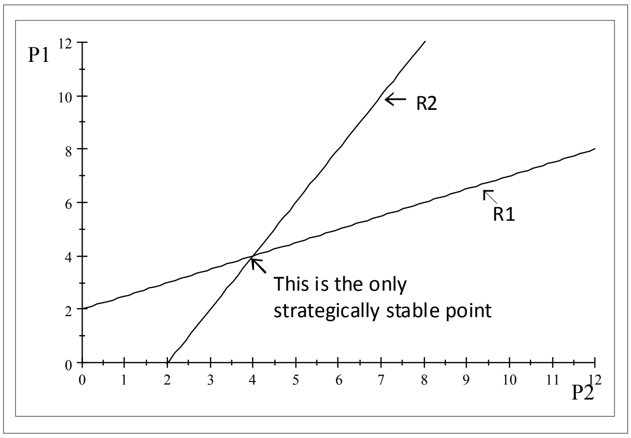

Figure 6: The strategic reaction functions of firms in Bertrand competition

The intersection point in the above diagram is the only strategically stable point. Any set of prices in the upper right quadrant will result in each firm wanting to set a lower price (based on their reaction functions). Similarly, any set of prices in the lower left quadrant would mean that each firm wants to set a higher price.

We modify the differentiated Bertrand model in the next section so as to capture the effects of localization by a foreign firm. We will use the augmented model to analyse real life situations where firms are faced with a choice of R&D localization.

2.2. The Differentiated Bertrand Model with a choice of localization

We adapt the two-firm differentiated Bertrand model to allow for the choice of localization by the foreign firm. First, we make some assumptions about the two firms.

Firm L is the representative/aggregate local firm in the host country. Firm L can also represent fully localized foreign firms. We can think of the rival firm as an aggregate firm with a substantially localized product.

Firm F is the foreign MNC and has a localization choice of γ. $\gamma \in (0,1]$. If γ is close to zero, there is no localization at all. If $\gamma = 1$, then there is full localization. Firm F has a second characteristic; ψ. ψ represents the brand disadvantage of Firm F, $\psi \in (0,1]$. $\psi =1$ denotes where the foreign product has no brand advantage over Firm L. As ψ approaches zero, the brand advantage becomes stronger.

We can write each firm's demands as follows:

$$q_L = A + \beta(\psi p_F - p_L)$$
$$q_F = A + \beta(\gamma p_L - p_F) \quad (4)$$

- ***A*** is the coefficient which represents the intercept of the linear demand
- $\boldsymbol{\beta}$ represents the innate differentiation between the products, this is stipulated by nature
- $\boldsymbol{\gamma}$ represents the amount of localization, increasing γ levels the playing ground between

the two firms.

- ψ represents the amount of brand disadvantage, $0< \psi <1$ signifies Firm F's brand disadvantage over Firm L. We assume that ψ is a fixed exogenous parameter which cannot be manipulated by Firm F.

Each firm's profit function is defined as $\pi_i = q_i p_i$. Without loss of generality, marginal costs are normalized to zero. We maximize profits by choosing the optimal choice of prices, this derives the reaction (or best response) functions of each firm:

$$R_L \Rightarrow p_L = \frac{1}{2\beta}A + \frac{\psi}{2}p_F$$
$$R_F \Rightarrow p_F = \frac{1}{2\beta}A + \frac{\gamma}{2}p_L \qquad (5)$$

As γ is the localization choice made by Firm F, we need to consider how this choice affects prices in the market. A change in γ introduces shifts in the reaction function of Firm F, the graph below shows R_F plotted with different values of γ.

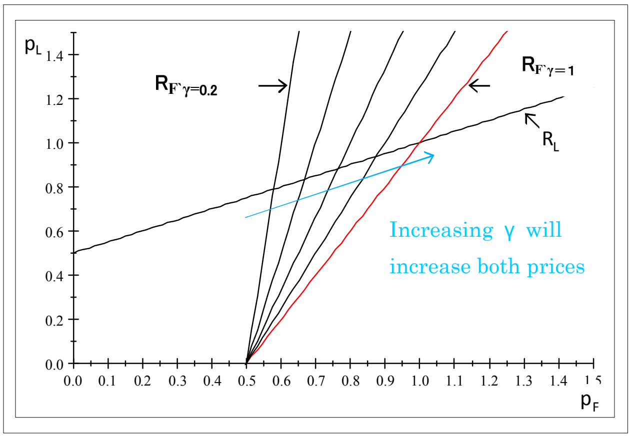

Figure 7: Reaction functions as γ changes

As γ increases, the reaction function of Firm F pivots to the right, this causes the equilibrium in prices to increase for both firms. Both firms benefit from lesser competition, firms' best reactions if γ increases are to increase prices.

Solving the two reaction functions explicitly, we can derive the equilibrium prices as:

$$p_L^* = A\frac{2+\psi}{\beta(4-\gamma\psi)}$$
$$p_F^* = A\frac{2+\gamma}{\beta(4-\gamma\psi)} \qquad (6)$$

We can substitute these prices into the demand functions to derive the quantities produced in equilibrium:

$$q_L^* = A\frac{2+\psi}{4-\gamma\psi}$$
$$q_F^* = A\frac{2+\gamma}{4-\gamma\psi} \qquad (7)$$

This set of equilibrium quantity and prices gives us the following (optimized) profits:

$$R_L^* = A^2\frac{(2+\psi)^2}{\beta(4-\gamma\psi)^2}$$
$$R_F^* = A^2\frac{(2+\gamma)^2}{\beta(4-\gamma\psi)^2} \qquad (8)$$

The effect of reducing ψ is to reduce the local advantage of Firm L. This means that it becomes easier for Firm F to do better than Firm L with a lower amount of γ. Using A=1 and β=0.5, we plot the difference in revenue (R^*_L-R^*_F) to analyze the effects of ψ in the graph below:

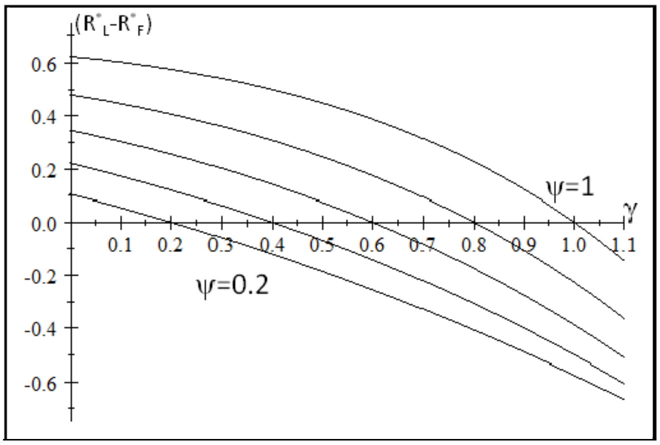

Figure 8: Firm L's revenue advantage over Firm F given ψ

As ψ gets smaller, the amount of γ needed to achieve equal revenue between the two firms become smaller. The derivative of R_F^* gives us the marginal revenue of firm F and can be calculated as:

$$R_F^*{}'(\gamma) = \frac{A^2}{\beta}\frac{4(2+\psi)(2+\gamma)}{(4-\gamma\psi)^3} \tag{9}$$

The marginal revenue of Firm F is increasing as long as γψ is lesser than 4. Since increases in γ are beneficial to Firm F in terms of prices, we are interested to see if there is an optimal amount of localization (γ) to choose. Assuming fixed numerical values for A and β, we can plot the revenue of Firm F against γ:

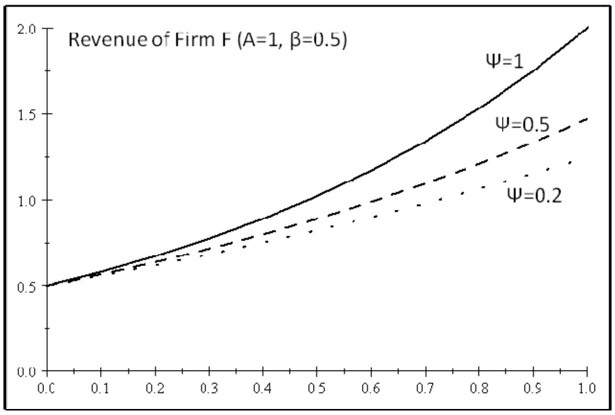

Figure 9: Revenue of Firm F given ψ

The revenue of Firm F is strictly increasing with γ, a larger γ means greater profits. We can also observe that a smaller ψ would reduce the profitability of increasing γ. However, as long as γ is costless to implement, there would be no reason not to choose γ=1. The structure of localization costs is an important factor in deciding the amount of localization to adopt. We will explore how different cost structures will affect the localization choice in the next section.

2.2.1. Positive costs of localization

If we allow costs of localization to be positive, we may observe cases where γ is close to zero. In the chart below outlines four examples of costs which would require different levels of γ to maximize profits. The dark blue lines represent revenue and the light blue lines represent costs, profits are maximized where the gap between the two is the biggest.

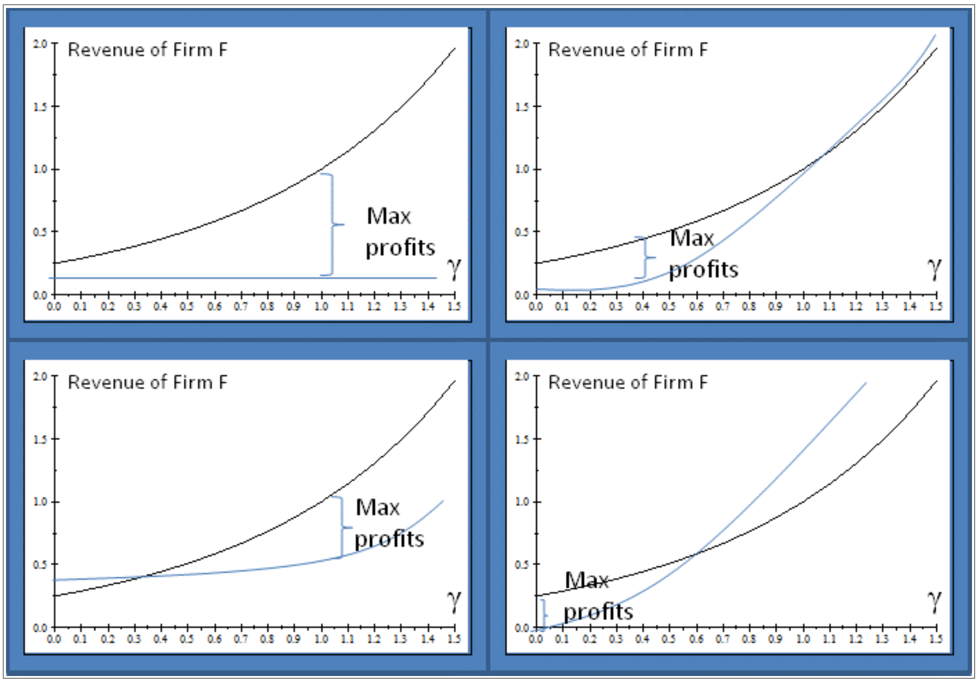

Figure 10: Four scenarios of revenue and costs

As seen in the above diagram, the maximum revenue-cost gap may exist at different values of γ. We can summarize these possibilities to three general cases:

1. Costs are too high or increase at too high a rate: Firm F always chooses γ=0
2. Costs are low or increase at a slow rate: Firm F always chooses γ=1
3. Costs are intermediate: The γ which optimizes profits is between 0 and 1

Understanding the composition of this cost is important in helping us understand why some firms invest in much more localization than others. The cost of localization can be made up of two types:

1. Explicit cost: the direct monetary costs associated with building and running research and development facilities

2. Implicit costs: These are non-monetary costs which are usually associated with risk attitudes. Implicit costs may depend on a few factors: (i) the local country; it may be riskier to set up an R&D plant in particular countries, (ii) Home country of the firm: certain country have specific cultures which are more conservative in investment attitudes, (iii) industry

factors: certain products have more valuable intellectual property rights (IPRs) and (iv) level of patience: some firms or managers prefer to take a wait and see attitude.

If costs are linear ($C(\gamma)=c\gamma$), profits are still maximized (or losses are minimized) at where $\gamma = 1$. However if marginal costs are increasing at a fast rate, the revenue-cost gap narrows as γ grows. In this case, the firm should choose zero localization. In the next section, we analyze the case of a simple increasing cost function which is commonly adopted in economics.

2.2.2. A quadratic cost function

We assume the following functional form for costs of localization: $C(\gamma) = \frac{\gamma^2}{2}$. This function has two useful mathematical properties:

1) Its derivative is γ. Marginal costs of localization = γ.
2) It is twice differentiable and increasing at an increasing rate.

In order to maximize profits, we simply equate marginal revenue to marginal costs. Recall the marginal revenue function derived from the base model with zero localization costs:

$$R_F^{*}{}'(\gamma) = \frac{A^2}{\beta}\frac{4(2+\psi)(2+\gamma)}{(4-\gamma\psi)^3} \qquad (10)$$

We set marginal revenue equal to marginal costs to find the profit maximizing amount of γ. There is also an additional constraint where γ must lie between zero and one. In the diagram below, we see an example of where the optimum γ lies within this constraint and maximizes profits at a point where $\gamma<1$.

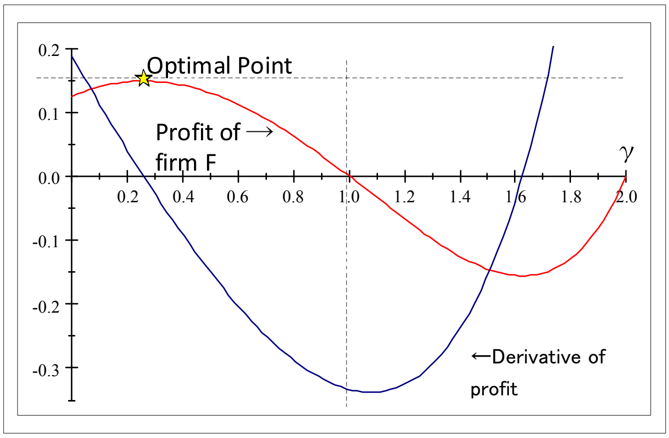

Figure 11: Example of optimal γ (1)

However, the profit maximizing γ may not lie within this constraint. In that case, we need to find the γ which gives the largest possible profits with the constraints. In the diagram below, we demonstrate one such case, profits are largest where γ=1.

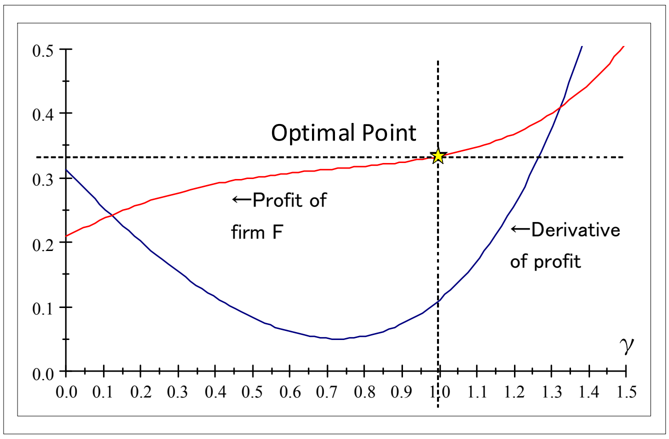

Figure 11: Example of optimal γ (2)

We can vary the values of ψ to analyze its effects on profits. In the graph below, we can see that decreasing ψ will decrease the profit maximizing level of γ (the highest point on the function). A strong brand advantage (of low ψ) means that Firm F will choose less localization.

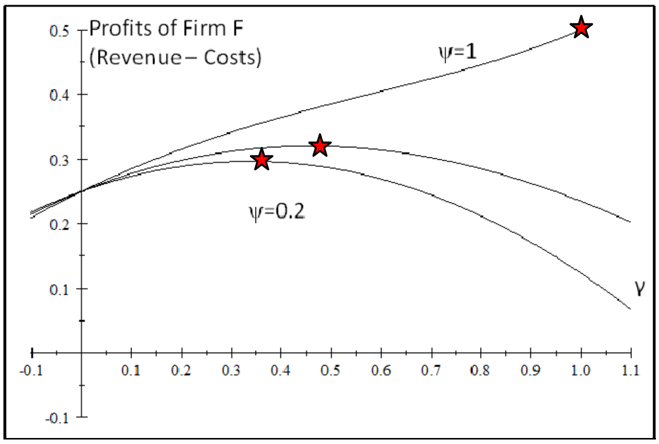

Figure 13: Optimal γ as ψ changes

We also observe that when ψ is high, profits may be maximized by choosing γ=1. As ψ approaches zero, the optimal γ should also approach zero. Thus if Firm F has an absolute brand advantage we may see cases where no amount of localization is adopted by Firm F. Due to the complexity of the solution to the optimal γ, a solution cannot be analytically obtained. Thus to analyze the effects of γ and ψ, we adopt numerical simulation methods in the next section.

2.2.3. Numerical Simulation

We set $Z = \frac{A^2}{\beta}$ and run numerical simulations for different levels of ψ. Matlab was used to calculate the optimal γ for incremental levels of Z and ψ (see Appendix for the full simulations results). Using values obtained from the numerical simulation, we plot the optimal γ against Z. We can use this graph to assess how the optimal γ reacts to changes in Z and ψ:

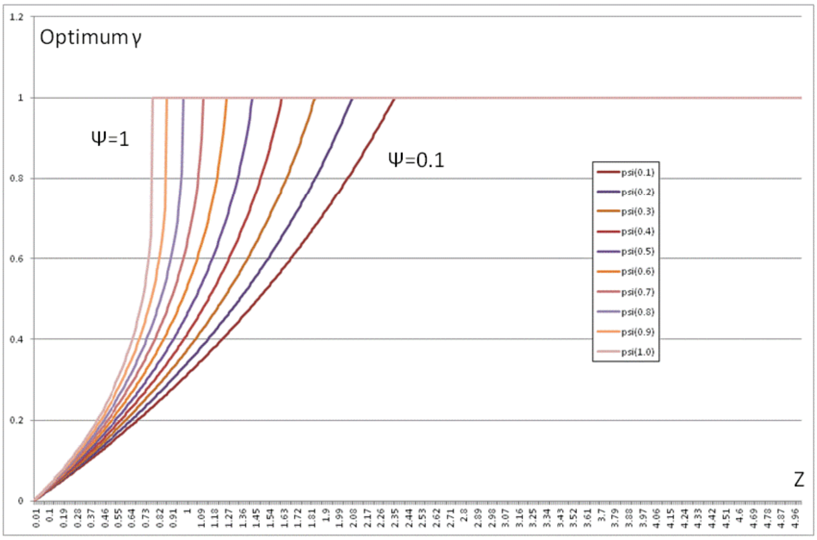

Figure 14: Optimum γ in response to change in Z and ψ

The optimum level of γ is restricted at a maximum of 1. The graph above shows that as Z increases, the level of γ needed to maximize profits is also increasing. When ψ is smaller, γ<1 can exist for a wider range of Z. In the next graph, we calculate the optimal profits and plot that against Z, using different values of ψ:

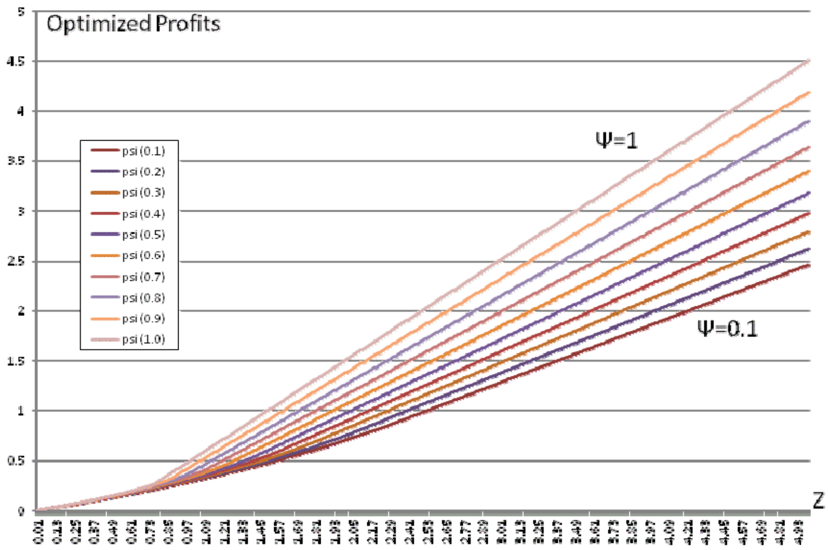

Figure 15: Optimized profits as it changes with Z and ψ

We can observe from the above graph that a lower ψ results in a lower level of profits. This is an interesting observation and we can attribute this to the brand differentiation effect. When ψ is small, the brand differentiation between firms is larger. Consumers are loyal to particular brands but this means that firms are less likely to attract other buyers with lower prices.

2.2.3.1. How do we interpret Z?

Recall that $Z = \frac{A^2}{\beta}$. Thus, Z is increasing with A but decreasing with β. The demand functions are of the following form:

$$q_F = A + \beta(\gamma p_L - p_F)$$
$$q_L = A + \beta(\psi p_F - p_L) \qquad (11)$$

A represents the maximum demand in the market or the amount which would have been bought if the product was free. Large *A* means a larger market size. When *A* is large (Z is large), it is more likely that γ=1 will be chosen as the potential profits are larger. If ψ is small (brand advantage is high), firms may choose γ<0 even if the market size (*A*) is larger.

β is a factor of both own price elasticity and cross price elasticity for the firm. A large β would mean that the product has an inelastic demand. Thus when demand is inelastic, Z will be smaller, it will be likelier that γ<1. Even if ψ is large (brand advantage is low), firms are more likely to choose γ<1 if demand is elastic.

Whether the foreign firm will choose full localization (γ=1) or partial localization (γ<1), depends on a combination of both Z and ψ. We will discuss the real life implications of this result in Section 4. Prior to that, we will discuss one more factor which may affect the localization choice. This is the firm's risk attitude.

2.2.4. Incorporating Risk Attitude

The risk attitude of the foreign is a determining factor in the perception of the costs of localization. Risk in this context can be associated with potential problem such as the leakage of technology or the loss in production quality. To incorporate this into our model, we assume that the cost function is $C(\gamma) = r\frac{\gamma^2}{2}$. r represents the risk attitude of the firm. A large r means higher costs of localization due to the high risk attitudes of the firm. The second order conditions in the previous section (where MR=MC) becomes:

$$Z\frac{4(2+\psi)(2+\gamma)}{(4-\gamma\psi)^3}=r\gamma \qquad (12)$$

Z can be replaced with $Z_r = Z/r$, the equations can be solved the same way as section 2.2.2. In addition, we can derive an interpretation for risk attitude. As r gets larger, Z_r gets smaller; this would mean that the optimal γ would be more likely to be below 1. Thus, if a firm has high risk attitude, it will engage in less localization. As r approaches infinity, the optimal γ will approach zero. However, we can see that some amount of localization is strategically optimal for the firm as long as r is not infinite.

Using the results of our model, we identified some determining factors which will affect the amount of localization. We analyze the real life implications of these factors in the next section.

3. Analysis and Discussion

3.1. Empirical evidence from Ito and Wakasugi (2007)

In Ito and Wakasugi (2007)'s paper titled: "What factors determine the mode of overseas R&D by multinationals? Empirical evidence", an empirical study was conducted on the factors which would increase R&D in foreign countries. Ito and Wakasugi (2007) distinguished between two types of R&D, "Support oriented" and "Knowledge Sourcing". The latter is similar to our description of a larger amount of localization. In the paper, foreign research labs were used as a proxy for "Knowledge Sourcing" R&D. Three factors which were found to have a statistically significant and positive effect on both types of R&D are:

1. Export propensity of affiliate firms. The more likely affiliate firms in the host country are to export, the more likely R&D will be conducted in the host country.

2. Abundance of human resource, we can associate this factor with a decrease in r (in our model). If human resources (talent) are easily available, the risks of localization should be perceived to be lower.

3. Accumulation of technical knowledge, this has a similar effect of decreasing r (in our model). A strong technical knowledge base in the host country would reduce the perceived risk.

4. Intellectual property rights (IPR) protection was found to have a positive effect on "knowledge sourcing" R&D only. This factor also decreases r (in our model).

Thus, we can see that the idea of risk attitudes in our model is supported empirically. Previous empirical studies such as Ito and Wakasugi (2007) analyzed the effects of how the marcoeconomic factors of the host country could affect the amount of R&D a foreign firm is willing to conduct there. However, there is a lack of analysis on how market structure factors (such as A, β and ψ) will affect foreign R&D decisions. If data is available, it would be interesting to test the relationship between the localization choice and variables such as market size, product differentiation, brand and other market related variables.

3.2. Policy implications from our results

In this section, we discuss four main results (in relation to the amount of localization γ) arising from our model. The four main results are, A the market size, β the amount of product differentiation, ψ the brand disadvantage and r the risk attitude of the foreign firm. We will discuss how these results can be used to explain real life scenarios and the possible policy implications that can arise. The government of a developing country may wish to attract R&D from developed countries as this can stimulate the economy; we discuss how this can be done in relation to each factor.

3.2.1. A, the market size

Given that all other variables remain constant, a larger A would result in a larger amount of γ (localization) chosen by the foreign firm. A large local market means a huge amount of potential profits to be made from its consumers. We observe that real life firms (such as GE) with foreign R&D subsidiaries tend to locate it in large countries with huge potential demand such as China, India, the United States of America and Europe.

As such, a potential policy which an emerging country's government can take in order to encourage inflow of R&D is to change expectations of market size, this is supported by Kuemmerle (1999)'s empirical study. This can be achieved with foreign policy such as inviting delegates from MNCs to visit the country, providing information sessions (such as trade fairs) in foreign countries and setting up outreach offices in cities where these large companies are located. One example of such outreach offices is Singapore's Economic Development Board (EDB). The EDB maintains 19 international offices in 10 countries such as China, France, Germany, Japan, United States and United Kingdom. These offices function to solicit foreign companies and attract them to set up offices in Singapore.

3.2.2. β, the amount of product differentiation

A smaller β means that the product has more elastic demand. These are products where consumers may easily switch to other substitutes if prices increase. Thus, price competition is not desirable in such a market and firms have to use product characteristics to get an advantage. A smaller β would mean that a larger γ will be chosen as products are not sufficiently differentiated and more localization is necessary in order to avoid aggressive price competition. Electronic and high technology goods such as cameras tend to be highly differentiated; we can observe that international brands (such as Cannon and Olympus) in this market do not localize their products for certain countries. Localization in this type of market tends to be minimal, such as the changing of language interface.

A policy which can be taken by governments wishing to attract foreign R&D would be to target firms competing in markets where goods are less differentiated. These are markets where consumers are more price sensitive and have less brand loyalty. Localization will give these firms more product advantage without having to go into price competition.

3.2.3. r the risk attitude of the foreign firm.

A larger r means a higher expectation of risks associated with localization. This would cause the optimal γ chosen to be lower. Since firms associate localization as risky, they would choose less of it. This is observed in more conservative firms taking the wait and see attitude.

Countries wishing to attract foreign R&D could try to improve intellectual property rights (IPR) protection in their country. These laws should be made stricter and seriously enforced. Another policy approach would be to increase the base of the local scientific community; this can be done through improving education and encouraging research collaborations with foreign academic institutions. We can observe that large emerging countries such as China and India have taken this approach.

3.2.4. ψ, the brand disadvantage

ψ has two effects in our model. Firstly, it has a direct effect of increasing the optimal γ chosen if all other factors are kept constant. This means if the foreign firm experience a decrease in brand advantage over the local competitor it will choose more localization.

Secondly, it can increase the range for which a firm may want to choose a γ lesser than 1, given the above three factors. Comparing two foreign firms, the firm with the larger brand advantage will tend to choose less localization. We can use this characteristic to offer an explanation for why Japanese firms may choose not to localize their products or setup foreign R&D subsidiaries. They enjoy substantial brand advantage over their rivals and have no need to engage in localization to remain competitive.

Thus, we can also conclude that it would be quite hard for a country wishing to attract firms with high brand advantages to setup a R&D facility in their country. If the technology and potential spillovers are large and beneficial to the country, the government should consider providing additional incentives to attract these firms. This can be done via tax incentives, or the infrastructure such as the industrial parks found in China.

3.3. Explaining the localization decisions of Japanese firms

In this section we try to explain the puzzle of why Japanese firms tend to be lagging in international R&D by using the results of our model. In our opinion, one strong reason why foreign R&D is not conducted by Japanese firms is the fact that these firms enjoy very strong brand advantages and they do not feel that it is necessary. They do not view local competitors as large threats. This conjecture is further supported through several interviews with officers of Japanese MNCs[3].

Another factor which may also influence the localization decision could be cultural aspects. Japanese culture is very different from that of western firms and the reluctance to setup foreign R&D subsidiaries may be due to this reason. Firms may feel that keeping R&D within the country can help to protect jobs of their current employees. Firms are also more willing to work with their own suppliers and the foreign supply chain may be seen as unreliable. Fukao et. al. (2006) conducted an empirical analysis on Japanese firms and their procurement process when operating in foreign countries, it was found that: “Japanese automobile assemblers in China did not choose suppliers based on their current labor productivity levels and assembler-supplier transactions were more closed for Japanese firms…” It was also mentioned in Fukao et. al. (2006) that “Japanese-owned suppliers tended to supply only a small number of assemblers and these tended to be predominantly Japanese.”

[3]These interviews were made possible with the assistance of JCER. I would like to express my gratitude to Shigesaburo Kabe and Go Yamada for making this possible. Unfortunately due to the sensitivity of this information, the interviewees and the firms they represent have to remain confidential.

The perception of the risk of knowledge leakage may also be another factor. Japanese firms may view the risk of R&D leakage as more costly, many Japanese MNCs are producers of high technology products and this adds to the risk. Firms may fear losing control over product decisions to overseas subsidiaries. Quality conscious firms may also see the potential risk of quality reduction as costly. Certain host countries may also be viewed as riskier locations as intellectual property rights protection is not strong. The startup costs in these countries may also be perceived as high, this may be due to language and cultural differences or the existence of a large amount of government red tape.

3.4. Discussion

The markets of China, India and other similar emerging markets are substantially large. There should be high incentives to localize in these markets as they represent large potential demand. Many western firms have made this move and have R&D subsidiaries in these countries. We outlined the various reasons in the previous section as to why Japanese firms may not have done likewise.

However, with the changing global landscape, we feel that Japanese firms may wish to act strategically or preemptively to prevent the rise of their competitors in their respective global markets. Setting up R&D subsidiaries has a strategic value in that they can absorb the most talented researchers in the country. This will block access to the best talents for the local firms who may potentially be strong competitors in the future. Thus, the Japanese government may also wish to play a role in encouraging firms to venture abroad with their research and development.

The Japanese government may have an incentive to encourage Japanese MNCs to engage in more overseas R&D due the following reasons:

1. This can preempt the rise of R&D of rival firms of Japanese MNCs and ensure Japan's position as brand leaders. This could help ensure continuous growth in Japan.

2. Knowledge spillover into Japan may stimulate the scientific community. This synergy can

encourage introduce new ideas and the creation of new and innovative products. This will further reinforce the brand advantage of the firm.

3. Successful entry into huge markets means large capital flows back to Japan. Being able to create products which are highly demanded by emerging markets would be very profitable. Although Japanese products are highly desired globally in the current market, this may not be so in the future when rival firm evolve.

We suggest a few measures which the government can take to encourage foreign R&D. The Japanese government can share risks with its firms by cooperating on the R&D venture. This can be done in the form of subsidies or joint ventures. The government can initiate research hubs in potential host countries. One example of this is the Suzhou Industrial Park, a joint venture between China and Singapore. Since this would be a collective effort of many firms, individual firms may find it less risky as other firms are in it together. The Japanese government can also be proactive in global intellectual property rights protection. IPR protection can be reinforced between countries with bilateral or regional trade agreements.

4. Conclusion

We set out in the report to analyze the localization choice from a microeconomic theory perspective. We did this with a modified differentiated Bertrand model aimed at examining the effect of competition to the localization choice. Simplifying assumptions were made so as to isolate important factors for analytical purposes. Although many assumptions were made, the results are consistent with intuition and reality. These results should not be interpreted on face value. Rather, one should take away the relative effects of each factor and its impact on the localization choice. Due to the simple structure of the model, it does not address two issues. First, the model does not take into account potential implications in the international market. Second, the model is not dynamic and does not allow for accumulated effects of localization. We discuss two potential extensions that can deal with these issues in the next section.

4.1. Considering the international market

We only consider the foreign market in this model. If we consider the international market, a factor which we should consider is the potential spillover of research back into the home and international markets. Innovation in the host country can potentially find a market in other countries; we see an example of this with GE's medical imaging equipment. These innovations may also provide a stimulus to researchers in other subsidiaries and information can be shared to foster more innovation. Thus, an extension which could be made to this model is to consider the effects of research spillovers and positive externalities which could result from localization.

4.2. Dynamics

The model we presented is deterministic in nature and shows how markets and firms fare given the decisions that are made by its managers. However it is not dynamic and does not consider the evolution of the market. The investment in localization in one period will spill over to the next, market sizes may evolve as the product becomes established. These factors are not considered here. A natural extension would be to consider the dynamic effects and examine how the path of localization changes over time. We may expect this to depend on the initial conditions of the firm (such as whether a brand advantage exists before entry into the market). In such a scenario, a firm with higher brand advantage may not choose low localization if they consider the future impact on the competitiveness within the market.

4.3. Concluding Remarks

To remain competitive in this increasingly globalized world, continual innovation is very important for an international firm. To achieve this, firms can choose to start R&D subsidiaries in locations other than their home countries. This act of localization is increasingly important, not only because these locations are fast growing emerging markets, but also because of the strategic reason to pre-empt potential competitors emerging from the local market. However, we observe in real life that not all firms are jumping into the trend of localization.

We presented a model in this paper which uses four microeconomic factors to explain this phenomenon. We acknowledge that this model does not completely address the complex issue of the localization decision and suggested two potential extensions to be undertaken in future research.

5. Appendix – Numerical Simulation Results

	ψ=0.1		ψ=0.2		ψ=0.3		ψ=0.4		ψ=0.5	
Z	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**
0.01	0.002628969	0.002503451	0.002754926	0.002503788	0.002881009	0.002504141	0.003007223	0.002504511	0.003133577	0.002504896
0.02	0.005265902	0.005013823	0.005519748	0.005015179	0.005774099	0.005016601	0.006028985	0.005018087	0.006284438	0.005019639
0.03	0.007910842	0.007531149	0.008294531	0.007534215	0.008679368	0.00753743	0.009065427	0.007540794	0.009452778	0.00754431
0.04	0.010563827	0.01005546	0.011079339	0.010060936	0.011596916	0.010066682	0.012116691	0.010072699	0.0126388	0.010078991
0.05	0.013224901	0.012586788	0.01387424	0.012595385	0.014526842	0.012604411	0.015182923	0.01261387	0.015842707	0.012623768
0.06	0.015894105	0.015125166	0.016679299	0.015137603	0.017469247	0.01515067	0.018264271	0.015164375	0.019064706	0.015178725
0.07	0.018571481	0.017670625	0.019494586	0.017687633	0.020424234	0.017705515	0.021360883	0.017724282	0.022305009	0.01774395
0.08	0.021257071	0.020223198	0.022320167	0.020245518	0.023391907	0.020269	0.024472913	0.020293663	0.025563832	0.020319528
0.09	0.023950918	0.022782918	0.025156112	0.022811301	0.026372371	0.022841181	0.027600513	0.022872587	0.028841394	0.02290555
0.1	0.026653065	0.025349818	0.028002491	0.025385025	0.029365733	0.025422115	0.030743842	0.025461127	0.032137918	0.025502106
0.11	0.029363555	0.027923931	0.030859373	0.027966735	0.032372101	0.028011858	0.033903058	0.028059355	0.035453634	0.028109287
0.12	0.032082433	0.030505291	0.03372683	0.030556474	0.035391584	0.030610467	0.037078323	0.030667345	0.038788772	0.030727185
0.13	0.034809741	0.033093931	0.036604934	0.033154287	0.038424293	0.033218002	0.040269802	0.033285171	0.042143569	0.033355897
0.14	0.037545525	0.035689886	0.039493757	0.035760219	0.041470341	0.03583452	0.043477661	0.035912909	0.045518267	0.035995516
0.15	0.040289829	0.038293188	0.042393372	0.038374316	0.04452984	0.038460081	0.046702071	0.038550634	0.048913111	0.038646139
0.16	0.043042698	0.040903874	0.045303854	0.040996623	0.047602907	0.041094745	0.049943203	0.041198425	0.052328353	0.041307867
0.17	0.045804177	0.043521976	0.048225277	0.043627187	0.050689658	0.043738572	0.053201233	0.043856359	0.055764247	0.043980797
0.18	0.048574313	0.046147531	0.051157717	0.046266053	0.053790211	0.046391623	0.05647634	0.046524515	0.059221055	0.046665033
0.19	0.051353151	0.048780573	0.05410125	0.04891327	0.056904686	0.049053961	0.059768705	0.049202975	0.062699043	0.049360676
0.2	0.054140738	0.051421137	0.057055954	0.051568884	0.060033206	0.051725647	0.063078511	0.051891819	0.066198481	0.052067832
0.21	0.056937121	0.054069259	0.060021907	0.054232944	0.063175893	0.054406746	0.066405947	0.054591129	0.069719647	0.054786607
0.22	0.059742347	0.056724975	0.062999187	0.056905496	0.066332871	0.057097321	0.069751202	0.05730099	0.073262823	0.057517109
0.23	0.062556463	0.059388321	0.065987874	0.059586591	0.069504268	0.059797435	0.07311447	0.060021485	0.076828298	0.060259447
0.24	0.065379517	0.062059333	0.068988049	0.062276277	0.072690213	0.062507156	0.076495949	0.062752701	0.080416366	0.063013732
0.25	0.068211558	0.064738048	0.071999792	0.064974604	0.075890834	0.065226548	0.079895838	0.065494724	0.084027327	0.065780079
0.26	0.071052635	0.067424502	0.075023188	0.067681621	0.079106265	0.067955679	0.083314342	0.068247642	0.087661488	0.068558601
0.27	0.073902796	0.070118733	0.078058318	0.070397379	0.082336638	0.070694616	0.086751668	0.071011545	0.091319162	0.071349416
0.28	0.076762091	0.072820777	0.081105266	0.073121929	0.08558209	0.073443426	0.090208028	0.073786522	0.095000671	0.074152642
0.29	0.079630569	0.075530673	0.084164119	0.075855322	0.088842758	0.07620218	0.093683635	0.076572666	0.09870634	0.076968399
0.3	0.082508282	0.078248458	0.087234961	0.07859761	0.092118782	0.078970946	0.09717871	0.079370069	0.102436505	0.079796812
0.31	0.08539528	0.08097417	0.09031788	0.081348845	0.095410303	0.081749795	0.100693474	0.082178826	0.106191506	0.082638003
0.32	0.088291614	0.083707848	0.093412964	0.08410908	0.098717466	0.084538799	0.104228153	0.084999031	0.109971693	0.085492101
0.33	0.091197335	0.08644953	0.096520301	0.086878367	0.102040414	0.08733803	0.10778298	0.087830782	0.113777422	0.088359233
0.34	0.094112496	0.089199255	0.099639981	0.089656761	0.105379297	0.09014756	0.111358188	0.090674177	0.11760906	0.091239532
0.35	0.097037148	0.091957063	0.102772095	0.092444316	0.108734264	0.092967464	0.114954017	0.093529316	0.121466979	0.09413313
0.36	0.099971344	0.094722992	0.105916736	0.095241086	0.112105467	0.095797816	0.11857071	0.096396299	0.12535156	0.097040163

	ψ=0.1		ψ=0.2		ψ=0.3		ψ=0.4		ψ=0.5	
Z	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**
0.37	0.102915139	0.097497083	0.109073995	0.098047127	0.115493062	0.098638691	0.122208517	0.099275228	0.129263196	0.099960769
0.38	0.105868585	0.100279376	0.112243967	0.100862494	0.118897203	0.101490167	0.125867689	0.102166209	0.133202285	0.102895089
0.39	0.108831736	0.10306991	0.115426748	0.103687243	0.122318051	0.104352319	0.129548485	0.105069345	0.137169237	0.105843265
0.4	0.111804647	0.105868727	0.118622432	0.106521431	0.125755767	0.107225227	0.133251168	0.107984745	0.141164472	0.108805444
0.41	0.114787373	0.108675867	0.121831118	0.109365116	0.129210515	0.11010897	0.136976004	0.110912516	0.145188417	0.111781772
0.42	0.117779969	0.111491372	0.125052903	0.112218355	0.13268246	0.113003627	0.140723267	0.113852768	0.149241514	0.114772402
0.43	0.120782492	0.114315282	0.128287888	0.115081205	0.136171772	0.11590928	0.144493235	0.116805614	0.153324212	0.117777487
0.44	0.123794998	0.117147641	0.131536172	0.117953728	0.139678623	0.118826011	0.148286191	0.119771167	0.157436972	0.120797183
0.45	0.126817543	0.119988489	0.134797858	0.12083598	0.143203186	0.121753902	0.152102426	0.122749542	0.161580267	0.123831649
0.46	0.129850184	0.12283787	0.138073049	0.123728024	0.146745638	0.124693038	0.155942234	0.125740856	0.165754583	0.126881048
0.47	0.132892981	0.125695825	0.141361849	0.126629918	0.15030616	0.127643504	0.159805915	0.128745228	0.169960415	0.129945545
0.48	0.13594599	0.128562399	0.144664363	0.129541725	0.153884932	0.130605385	0.163693777	0.131762777	0.174198275	0.13302531
0.49	0.139009271	0.131437634	0.147980699	0.132463506	0.15748214	0.133578769	0.167606134	0.134793627	0.178468685	0.136120513
0.5	0.142082883	0.134321574	0.151310963	0.135395324	0.161097974	0.136563744	0.171543305	0.137837902	0.182772181	0.13923133
0.51	0.145166886	0.137214262	0.154655267	0.13833724	0.164732623	0.1395604	0.175505616	0.140895728	0.187109315	0.142357939
0.52	0.14826134	0.140115744	0.158013719	0.14128932	0.168386282	0.142568826	0.179493401	0.143967233	0.191480653	0.145500524
0.53	0.151366306	0.143026064	0.161386433	0.144251627	0.172059149	0.145589115	0.183506999	0.147052548	0.195886774	0.148659269
0.54	0.154481845	0.145945267	0.164773521	0.147224225	0.175751424	0.148621359	0.187546759	0.150151804	0.200328276	0.151834364
0.55	0.157608019	0.148873397	0.168175098	0.150207181	0.179463311	0.151665652	0.191613035	0.153265137	0.204805772	0.155026003
0.56	0.160744891	0.151810502	0.17159128	0.153200561	0.183195018	0.154722089	0.195706189	0.156392683	0.209319891	0.158234384
0.57	0.163892523	0.154756626	0.175022185	0.15620443	0.186946755	0.157790766	0.199826592	0.159534582	0.213871282	0.161459707
0.58	0.167050979	0.157711816	0.178467932	0.159218858	0.190718737	0.160871782	0.203974622	0.162690973	0.21846061	0.164702178
0.59	0.170220324	0.160676118	0.181928641	0.16224391	0.194511181	0.163965234	0.208150667	0.165862002	0.223088561	0.167962009
0.6	0.17340062	0.16364958	0.185404435	0.165279658	0.198324309	0.167071223	0.212355121	0.169047814	0.227755841	0.171239412
0.61	0.176591935	0.16663225	0.188895436	0.168326169	0.202158346	0.170189851	0.216588389	0.172248558	0.232463174	0.174534608
0.62	0.179794333	0.169624174	0.19240177	0.171383514	0.206013521	0.17332122	0.220850884	0.175464384	0.237211308	0.177847821
0.63	0.183007881	0.1726254	0.195923564	0.174451765	0.209890068	0.176465434	0.22514303	0.178695446	0.242001014	0.18117928
0.64	0.186232645	0.175635978	0.199460945	0.177530992	0.213788224	0.1796226	0.229465259	0.181941901	0.246833084	0.184529218
0.65	0.189468694	0.178655957	0.203014044	0.180621268	0.217708229	0.182792824	0.233818013	0.185203907	0.251708336	0.187897876
0.66	0.192716095	0.181685384	0.206582992	0.183722666	0.22165033	0.185976214	0.238201746	0.188481627	0.256627613	0.191285498
0.67	0.195974918	0.184724311	0.210167923	0.18683526	0.225614776	0.18917288	0.242616922	0.191775225	0.261591784	0.194692335
0.68	0.19924523	0.187772786	0.213768971	0.189959125	0.229601821	0.192382934	0.247064016	0.195084869	0.266601747	0.198118643
0.69	0.202527103	0.19083086	0.217386273	0.193094336	0.233611725	0.19560649	0.251543513	0.198410729	0.271658428	0.201564685
0.7	0.205820607	0.193898585	0.221019968	0.196240969	0.23764475	0.19884366	0.256055912	0.20175298	0.276762782	0.205030729
0.71	0.209125814	0.19697601	0.224670196	0.199399101	0.241701166	0.202094562	0.260601724	0.205111797	0.281915798	0.208517051
0.72	0.212442794	0.200063188	0.2283371	0.202568811	0.245781246	0.205359313	0.265181471	0.208487362	0.287118497	0.212023932

Z	Optimal γ	Optimized Profits	Optimal γ	Optimized Profits	Optimal γ	Optimized Profits	Optimal γ	Optimized Profits	Optimal γ	Optimized Profits
0.73	0.215771621	0.203160171	0.232020823	0.205750176	0.249885267	0.208638033	0.26979569	0.211879858	0.292371935	0.215551661
0.74	0.219112368	0.20626701	0.235721511	0.208943276	0.254013512	0.211930843	0.27444493	0.215289472	0.297677202	0.219100534
0.75	0.222465108	0.209383759	0.239439313	0.212148192	0.258166272	0.215237866	0.279129755	0.218716395	0.303035429	0.222670855
0.76	0.225829917	0.212510471	0.243174378	0.215365004	0.262343839	0.218559226	0.283850744	0.22216082	0.308447786	0.226262935
0.77	0.229206868	0.2156472	0.246926858	0.218593795	0.266546514	0.22189505	0.288608488	0.225622947	0.313915484	0.229877093
0.78	0.23259604	0.218793998	0.250696908	0.221834648	0.270774602	0.225245466	0.293403598	0.229102976	0.319439777	0.233513657
0.79	0.235997507	0.221950922	0.254484683	0.225087647	0.275028413	0.228610604	0.298236697	0.232601114	0.325021968	0.237172964
0.82	0.246276462	0.231482992	0.265955957	0.234920369	0.287947396	0.238795683	0.312970439	0.243206299	0.342129683	0.248290842
0.83	0.249727893	0.234680968	0.26981624	0.238222808	0.292307338	0.242221052	0.317962091	0.246779013	0.347957487	0.25204467
0.84	0.253192015	0.237889348	0.273695063	0.241537826	0.296694655	0.245661823	0.322995122	0.250370929	0.353850477	0.255823066
0.85	0.256668908	0.241108189	0.277592594	0.244865512	0.301109695	0.24911814	0.328070266	0.253982279	0.359810286	0.259626426
0.86	0.260158654	0.244337548	0.281509007	0.248205958	0.305552816	0.252590146	0.33318828	0.257613299	0.365838615	0.26345516
0.87	0.263661335	0.247577483	0.285444476	0.251559255	0.310024381	0.256077989	0.338349942	0.261264233	0.371937236	0.267309687
0.88	0.267177035	0.250828053	0.289399177	0.254925496	0.314524763	0.259581818	0.343556053	0.264935327	0.378107993	0.271190441
0.89	0.270705838	0.254089317	0.29337329	0.258304774	0.31905434	0.263101784	0.348807437	0.268626834	0.384352813	0.275097867
0.9	0.274247828	0.257361334	0.297366998	0.261697184	0.323613501	0.266638041	0.354104944	0.272339012	0.390673702	0.279032426
0.91	0.277803093	0.260644165	0.301380486	0.265102823	0.32820264	0.270190745	0.359449446	0.276072126	0.397072758	0.282994592
0.92	0.281371718	0.26393787	0.30541394	0.268521786	0.332822161	0.273760055	0.364841847	0.279826446	0.403552172	0.286984855
0.93	0.284953791	0.267242509	0.309467551	0.271954173	0.337472477	0.277346133	0.370283074	0.283602246	0.410114237	0.291003721
0.94	0.288549401	0.270558145	0.313541513	0.275400083	0.342154009	0.280949142	0.375774086	0.28739981	0.41676135	0.295051711
0.95	0.292158636	0.273884839	0.31763602	0.278859616	0.346867187	0.284569249	0.381315871	0.291219426	0.423496024	0.299129366
0.96	0.295781586	0.277222654	0.321751273	0.282332874	0.351612451	0.288206624	0.386909449	0.29506139	0.430320893	0.303237244
0.97	0.299418343	0.280571653	0.325887472	0.285819959	0.356390251	0.291861438	0.392555873	0.298926005	0.43723872	0.307375922
0.98	0.303068998	0.283931898	0.330044824	0.289320976	0.361201046	0.295533867	0.398256231	0.302813579	0.444252407	0.311545998
0.99	0.306733645	0.287303456	0.334223535	0.292836031	0.366045306	0.299224089	0.404011646	0.30672443	0.451365006	0.315748093
1	0.310412376	0.290686389	0.338423817	0.296365229	0.370923511	0.302932285	0.409823279	0.310658883	0.458579726	0.319982848
1.01	0.314105287	0.294080764	0.342645885	0.299908678	0.375836154	0.306658639	0.415692332	0.314617271	0.465899949	0.32425093
1.02	0.317812473	0.297486645	0.346889956	0.303466489	0.380783736	0.310403339	0.421620047	0.318599935	0.473329239	0.32855303
1.03	0.321534031	0.3009041	0.351156252	0.30703877	0.385766771	0.314166575	0.42760771	0.322607226	0.480871361	0.332889867
1.04	0.325270058	0.304333195	0.355444997	0.310625635	0.390785788	0.317948541	0.433656652	0.326639501	0.488530291	0.337262187
1.05	0.329020652	0.307773998	0.359756419	0.314227197	0.395841323	0.321749436	0.439768252	0.33069713	0.496310238	0.341670768
1.06	0.332785914	0.311226576	0.36409075	0.31784357	0.40093393	0.325569459	0.445943941	0.334780489	0.504215661	0.346116418
1.07	0.336565943	0.314690998	0.368448225	0.32147487	0.406064173	0.329408815	0.452185201	0.338889968	0.512251288	0.350599982
1.08	0.340360842	0.318167334	0.372829085	0.325121215	0.41123263	0.333267713	0.45849357	0.343025964	0.520422144	0.355122338
1.09	0.344170712	0.321655653	0.377233571	0.328782724	0.416439895	0.337146365	0.464870644	0.347188887	0.528733574	0.359684404
1.1	0.347995658	0.325156025	0.38166193	0.332459518	0.421686575	0.341044987	0.471318082	0.351379157	0.537191273	0.364287142

	ψ=0.1		ψ=0.2		ψ=0.3		ψ=0.4		ψ=0.5	
Z	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**
1.11	0.351835784	0.328668522	0.386114415	0.336151719	0.426973293	0.344963799	0.477837607	0.355597206	0.545801319	0.368931554
1.12	0.355691197	0.332193216	0.390591279	0.33985945	0.432300687	0.348903024	0.48443101	0.35984348	0.554570206	0.373618691
1.13	0.359562002	0.335730178	0.395092783	0.343582836	0.437669412	0.352862893	0.491100156	0.364118435	0.563504893	0.378349658
1.14	0.363448308	0.339279481	0.39961919	0.347322005	0.44308014	0.356843636	0.497846986	0.368422544	0.572612843	0.383125609
1.15	0.367350225	0.342841199	0.404170768	0.351077085	0.448533558	0.360845493	0.504673521	0.37275629	0.58190208	0.387947764
1.16	0.371267862	0.346415406	0.408747789	0.354848207	0.454030376	0.364868703	0.511581867	0.377120173	0.591381245	0.3928174
1.17	0.375201331	0.350002178	0.413350531	0.358635502	0.459571316	0.368913515	0.518574224	0.381514709	0.601059672	0.397735868
1.18	0.379150744	0.353601589	0.417979276	0.362439104	0.465157126	0.37298018	0.525652886	0.385940427	0.610947459	0.402704593
1.19	0.383116216	0.357213716	0.42263431	0.366259148	0.470788571	0.377068953	0.532820248	0.390397875	0.621055561	0.407725079
1.22	0.395110141	0.368127164	0.436760089	0.377819318	0.487964677	0.389470573	0.554880179	0.403966348	0.652826508	0.423113557
1.23	0.399141011	0.37179093	0.441523248	0.381706524	0.49378674	0.393650455	0.562428591	0.408556558	0.663946633	0.428358062
1.24	0.403188527	0.375467804	0.446314206	0.385610877	0.499658591	0.397853809	0.570079464	0.413181518	0.675359055	0.433663377
1.25	0.407252812	0.379157865	0.451133282	0.389532526	0.505581136	0.402080926	0.577835963	0.417841894	0.687082813	0.439031693
1.26	0.411333987	0.382861195	0.455980799	0.393471618	0.511555304	0.406332102	0.585701413	0.422538378	0.699139047	0.444465359
1.27	0.415432177	0.386577877	0.460857088	0.397428304	0.51758205	0.41060764	0.593679311	0.427271684	0.711551347	0.449966902
1.28	0.419547507	0.390307992	0.465762483	0.401402739	0.523662361	0.414907848	0.60177334	0.432042555	0.724346157	0.455539053
1.29	0.423680105	0.394051624	0.470697327	0.405395077	0.529797251	0.419233043	0.609987377	0.43685176	0.737553285	0.461184768
1.3	0.427830098	0.397808858	0.475661967	0.409405475	0.535987767	0.423583548	0.618325517	0.441700098	0.75120653	0.466907259
1.31	0.431997617	0.401579778	0.480656759	0.413434094	0.542234986	0.427959693	0.626792081	0.446588401	0.765344467	0.472710034
1.32	0.436182792	0.405364471	0.485682064	0.417481095	0.54854002	0.432361815	0.635391639	0.45151753	0.780011443	0.47859694
1.33	0.440385757	0.409163022	0.49073825	0.421546643	0.554904016	0.436790262	0.644129032	0.456488385	0.795258858	0.484572214
1.34	0.444606645	0.41297552	0.495825691	0.425630905	0.561328157	0.441245386	0.653009387	0.461501901	0.81114683	0.490640553
1.35	0.448845592	0.416802053	0.500944769	0.42973405	0.567813665	0.445727549	0.662038151	0.466559053	0.827746408	0.496807197
1.36	0.453102736	0.420642709	0.506095875	0.43385625	0.574361801	0.450237122	0.671221111	0.471660859	0.845142562	0.503078032
1.37	0.457378214	0.424497579	0.511279406	0.437997679	0.58097387	0.454774484	0.680564428	0.476808382	0.8634383	0.509459729
1.38	0.461672168	0.428366754	0.516495765	0.442158514	0.587651217	0.459340023	0.690074673	0.482002732	0.882760519	0.515959919
1.39	0.46598474	0.432250325	0.521745365	0.446338934	0.594395236	0.46393414	0.699758862	0.487245072	0.903268508	0.522587439
1.4	0.470316072	0.436148385	0.527028628	0.450539122	0.601207368	0.46855724	0.709624502	0.49253662	0.925166824	0.529352658
1.41	0.474666311	0.440061028	0.532345983	0.454759264	0.608089104	0.473209744	0.719679639	0.497878652	0.94872555	0.536267953
1.42	0.479035604	0.443988347	0.537697867	0.458999546	0.615041989	0.477892079	0.729932913	0.503272511	0.974313965	0.543348418
1.43	0.483424099	0.447930438	0.543084726	0.463260159	0.622067621	0.482604687	0.74039362	0.508719607	1	0.550612245
1.44	0.487831947	0.451887398	0.548507018	0.467541298	0.629167659	0.487348018	0.751071786	0.514221425	1	0.557959184
1.45	0.492259299	0.455859324	0.553965207	0.47184316	0.636343821	0.492122538	0.761978248	0.51977953	1	0.565306122
1.46	0.496706311	0.459846313	0.559459767	0.476165943	0.64359789	0.496928721	0.773124745	0.525395577	1	0.572653061
1.47	0.501173138	0.463848465	0.564991183	0.480509852	0.650931717	0.501767057	0.784524031	0.531071315	1	0.58
1.48	0.505659938	0.46786588	0.570559951	0.484875093	0.658347224	0.506638048	0.79619	0.536808598	1	0.587346939

	ψ=0.1		ψ=0.2		ψ=0.3		ψ=0.4		ψ=0.5	
Z	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**
1.49	0.51016687	0.471898659	0.576166574	0.489261874	0.665846409	0.511542212	0.808137829	0.542609395	1	0.594693878
1.5	0.514694096	0.475946905	0.58181157	0.49367041	0.673431347	0.516480079	0.820384154	0.548475798	1	0.602040816
1.51	0.519241779	0.480010719	0.587495466	0.498100917	0.681104199	0.521452197	0.832947269	0.554410041	1	0.609387755
1.52	0.523810086	0.484090208	0.5932188	0.502553615	0.688867215	0.526459126	0.84584737	0.560414509	1	0.616734694
1.53	0.528399184	0.488185474	0.598982123	0.507028728	0.696722735	0.531501447	0.859106835	0.566491758	1	0.624081633
1.54	0.533009242	0.492296626	0.604785996	0.511526483	0.704673201	0.536579756	0.872750574	0.57264453	1	0.631428571
1.55	0.537640432	0.496423771	0.610630996	0.516047111	0.71272116	0.541694667	0.886806443	0.578875785	1	0.63877551
1.56	0.542292928	0.500567016	0.61651771	0.520590848	0.720869268	0.546846814	0.901305752	0.585188717	1	0.646122449
1.57	0.546966906	0.504726472	0.62244674	0.525157934	0.729120302	0.55203685	0.916283893	0.591586794	1	0.653469388
1.58	0.551662544	0.508902249	0.628418699	0.529748611	0.737477163	0.557265451	0.931781125	0.598073787	1	0.660816327
1.59	0.556380022	0.513094459	0.634434219	0.534363126	0.745942885	0.562533311	0.947843555	0.604653826	1	0.668163265
1.6	0.561119524	0.517303216	0.640493941	0.539001733	0.754520645	0.56784115	0.9645244	0.611331442	1	0.675510204
1.61	0.565881235	0.521528632	0.646598525	0.543664687	0.763213772	0.573189711	0.981885607	0.618111645	1	0.682857143
1.62	0.570665341	0.525770825	0.652748646	0.54835225	0.772025755	0.578579762	1	0.625	1	0.690204082
1.63	0.575472032	0.530029909	0.658944993	0.553064686	0.780960259	0.584012099	1	0.631944444	1	0.69755102
1.64	0.580301502	0.534306005	0.665188274	0.557802266	0.790021132	0.589487545	1	0.638888889	1	0.704897959
1.65	0.585153944	0.538599229	0.671479212	0.562565266	0.799212419	0.595006953	1	0.645833333	1	0.712244898
1.66	0.590029555	0.542909703	0.677818551	0.567353966	0.808538381	0.600571205	1	0.652777778	1	0.719591837
1.67	0.594928536	0.547237548	0.68420705	0.572168651	0.818003506	0.60618122	1	0.659722222	1	0.726938776
1.68	0.599851089	0.551582888	0.690645488	0.577009612	0.827612528	0.611837949	1	0.666666667	1	0.734285714
1.69	0.604797419	0.555945847	0.697134664	0.581877146	0.837370445	0.61754238	1	0.673611111	1	0.741632653
1.7	0.609767734	0.560326549	0.703675397	0.586771554	0.847282542	0.623295541	1	0.680555556	1	0.748979592
1.71	0.614762244	0.564725123	0.710268528	0.591693145	0.857354416	0.629098501	1	0.6875	1	0.756326531
1.72	0.619781163	0.569141697	0.716914919	0.596642231	0.867591994	0.634952373	1	0.694444444	1	0.763673469
1.73	0.624824706	0.5735764	0.723615455	0.601619133	0.878001571	0.640858318	1	0.701388889	1	0.771020408
1.74	0.629893094	0.578029363	0.730371042	0.606624176	0.888589836	0.646817545	1	0.708333333	1	0.778367347
1.75	0.634986547	0.58250072	0.737182614	0.611657694	0.899363907	0.652831317	1	0.715277778	1	0.785714286
1.76	0.640105292	0.586990605	0.744051128	0.616720025	0.910331375	0.658900955	1	0.722222222	1	0.793061224
1.77	0.645249556	0.591499152	0.750977568	0.621811516	0.921500344	0.66502784	1	0.729166667	1	0.800408163
1.78	0.65041957	0.5960265	0.757962945	0.626932519	0.932879483	0.671213418	1	0.736111111	1	0.807755102
1.79	0.65561557	0.600572786	0.765008296	0.632083395	0.944478081	0.677459206	1	0.743055556	1	0.815102041
1.8	0.660837792	0.605138152	0.772114691	0.637264512	0.956306111	0.683766796	1	0.75	1	0.82244898
1.81	0.666086478	0.609722739	0.779283227	0.642476244	0.968374302	0.690137861	1	0.756944444	1	0.829795918
1.82	0.671361872	0.61432669	0.786515036	0.647718977	0.980694222	0.696574163	1	0.763888889	1	0.837142857
1.83	0.676664223	0.618950151	0.793811279	0.652993101	0.993278369	0.70307756	1	0.770833333	1	0.844489796
1.84	0.681993781	0.623593268	0.801173154	0.658299016	1	0.709642075	1	0.777777778	1	0.851836735

Z	ψ=0.1		ψ=0.2		ψ=0.3		ψ=0.4		ψ=0.5	
	Optimal γ	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**
1.85	0.687350802	0.62825619	0.808601894	0.663637132	1	0.716216216	1	0.784722222	1	0.859183673
1.86	0.692735543	0.632939067	0.816098769	0.669007867	1	0.722790358	1	0.791666667	1	0.866530612
1.87	0.698148268	0.637642051	0.823665087	0.674411647	1	0.7293645	1	0.798611111	1	0.873877551
1.88	0.703589243	0.642365296	0.831302197	0.67984891	1	0.735938641	1	0.805555556	1	0.88122449
1.89	0.709058737	0.647108957	0.839011489	0.685320102	1	0.742512783	1	0.8125	1	0.888571429
1.9	0.714557024	0.651873191	0.846794399	0.69082568	1	0.749086925	1	0.819444444	1	0.895918367
1.91	0.720084382	0.656658158	0.854652407	0.696366113	1	0.755661066	1	0.826388889	1	0.903265306
1.92	0.725641094	0.661464019	0.862587041	0.701941878	1	0.762235208	1	0.833333333	1	0.910612245
1.93	0.731227445	0.666290937	0.87059988	0.707553466	1	0.76880935	1	0.840277778	1	0.917959184
1.94	0.736843726	0.671139077	0.878692554	0.713201379	1	0.775383492	1	0.847222222	1	0.925306122
1.95	0.742490232	0.676008605	0.886866748	0.718886131	1	0.781957633	1	0.854166667	1	0.932653061
1.96	0.748167262	0.680899692	0.895124205	0.724608249	1	0.788531775	1	0.861111111	1	0.94
1.97	0.753875119	0.685812508	0.903466728	0.730368272	1	0.795105917	1	0.868055556	1	0.947346939
1.98	0.759614113	0.690747226	0.911896181	0.736166754	1	0.801680058	1	0.875	1	0.954693878
1.99	0.765384556	0.695704021	0.920414497	0.742004262	1	0.8082542	1	0.881944444	1	0.962040816
2	0.771186767	0.700683072	0.929023676	0.747881379	1	0.814828342	1	0.888888889	1	0.969387755
2.01	0.777021067	0.705684558	0.937725791	0.753798703	1	0.821402484	1	0.895833333	1	0.976734694
2.02	0.782887786	0.71070866	0.946522993	0.759756845	1	0.827976625	1	0.902777778	1	0.984081633
2.03	0.788787256	0.715755563	0.955417511	0.765756437	1	0.834550767	1	0.909722222	1	0.991428571
2.04	0.794719816	0.720825454	0.964411661	0.771798126	1	0.841124909	1	0.916666667	1	0.99877551
2.05	0.800685809	0.725918521	0.973507845	0.777882574	1	0.84769905	1	0.923611111	1	1.006122449
2.06	0.806685584	0.731034956	0.982708563	0.784010467	1	0.854273192	1	0.930555556	1	1.013469388
2.07	0.812719496	0.736174953	0.992016411	0.790182506	1	0.860847334	1	0.9375	1	1.020816327
2.08	0.818787907	0.741338707	1	0.796398892	1	0.867421476	1	0.944444444	1	1.028163265
2.09	0.824891181	0.746526419	1	0.802631579	1	0.873995617	1	0.951388889	1	1.035510204
2.1	0.831029691	0.751738288	1	0.808864266	1	0.880569759	1	0.958333333	1	1.042857143
2.11	0.837203817	0.75697452	1	0.815096953	1	0.887143901	1	0.965277778	1	1.050204082
2.12	0.843413941	0.762235321	1	0.82132964	1	0.893718042	1	0.972222222	1	1.05755102
2.13	0.849660457	0.7675209	1	0.827562327	1	0.900292184	1	0.979166667	1	1.064897959
2.14	0.85594376	0.772831471	1	0.833795014	1	0.906866326	1	0.986111111	1	1.072244898
2.15	0.862264256	0.778167247	1	0.840027701	1	0.913440467	1	0.993055556	1	1.079591837
2.16	0.868622355	0.783528447	1	0.846260388	1	0.920014609	1	1	1	1.086938776
2.17	0.875018477	0.788915292	1	0.852493075	1	0.926588751	1	1.006944444	1	1.094285714
2.18	0.881453045	0.794328005	1	0.858725762	1	0.933162893	1	1.013888889	1	1.101632653
2.19	0.887926494	0.799766815	1	0.864958449	1	0.939737034	1	1.020833333	1	1.108979592

	ψ=0.1		ψ=0.2		ψ=0.3		ψ=0.4		ψ=0.5	
Z	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**
2.2	0.894439262	0.805231951	1	0.871191136	1	0.946311176	1	1.027777778	1	1.116326531
2.21	0.900991798	0.810723645	1	0.877423823	1	0.952885318	1	1.034722222	1	1.123673469
2.22	0.907584557	0.816242136	1	0.88365651	1	0.959459459	1	1.041666667	1	1.131020408
2.23	0.914218004	0.821787663	1	0.889889197	1	0.966033601	1	1.048611111	1	1.138367347
2.24	0.920892611	0.827360469	1	0.896121884	1	0.972607743	1	1.055555556	1	1.145714286
2.25	0.927608858	0.8329608	1	0.902354571	1	0.979181885	1	1.0625	1	1.153061224
2.26	0.934367233	0.838588907	1	0.908587258	1	0.985756026	1	1.069444444	1	1.160408163
2.27	0.941168236	0.844245044	1	0.914819945	1	0.992330168	1	1.076388889	1	1.167755102
2.28	0.948012374	0.849929468	1	0.921052632	1	0.99890431	1	1.083333333	1	1.175102041
2.29	0.954900163	0.855642441	1	0.927285319	1	1.005478451	1	1.090277778	1	1.18244898
2.3	0.961832129	0.861384226	1	0.933518006	1	1.012052593	1	1.097222222	1	1.189795918
2.31	0.968808808	0.867155094	1	0.939750693	1	1.018626735	1	1.104166667	1	1.197142857
2.32	0.975830747	0.872955316	1	0.94598338	1	1.025200877	1	1.111111111	1	1.204489796
2.33	0.982898501	0.878785171	1	0.952216066	1	1.031775018	1	1.118055556	1	1.211836735
2.34	0.990012638	0.884644938	1	0.958448753	1	1.03834916	1	1.125	1	1.219183673
2.35	0.997173735	0.890534903	1	0.96468144	1	1.044923302	1	1.131944444	1	1.226530612
2.36	1	0.896449704	1	0.970914127	1	1.051497443	1	1.138888889	1	1.233877551
2.37	1	0.902366864	1	0.977146814	1	1.058071585	1	1.145833333	1	1.24122449
2.38	1	0.908284024	1	0.983379501	1	1.064645727	1	1.152777778	1	1.248571429
2.39	1	0.914201183	1	0.989612188	1	1.071219869	1	1.159722222	1	1.255918367
2.4	1	0.920118343	1	0.995844875	1	1.07779401	1	1.166666667	1	1.263265306
2.41	1	0.926035503	1	1.002077562	1	1.084368152	1	1.173611111	1	1.270612245
2.42	1	0.931952663	1	1.008310249	1	1.090942294	1	1.180555556	1	1.277959184
2.43	1	0.937869822	1	1.014542936	1	1.097516435	1	1.1875	1	1.285306122
2.44	1	0.943786982	1	1.020775623	1	1.104090577	1	1.194444444	1	1.292653061
2.45	1	0.949704142	1	1.02700831	1	1.110664719	1	1.201388889	1	1.3
2.46	1	0.955621302	1	1.033240997	1	1.11723886	1	1.208333333	1	1.307346939
2.47	1	0.961538462	1	1.039473684	1	1.123813002	1	1.215277778	1	1.314693878
2.48	1	0.967455621	1	1.045706371	1	1.130387144	1	1.222222222	1	1.322040816
2.49	1	0.973372781	1	1.051939058	1	1.136961286	1	1.229166667	1	1.329387755
2.5	1	0.979289941	1	1.058171745	1	1.143535427	1	1.236111111	1	1.336734694

	ψ=0.6		ψ=0.7		ψ=0.8		ψ=0.9		ψ=1	
Z	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**
0.01	0.003260078	0.002505298	0.003386733	0.002505715	0.003513551	0.002506149	0.003640537	0.002506598	0.003767701	0.002507064
0.02	0.006540488	0.005021256	0.006797167	0.00502294	0.007054508	0.005024691	0.007312543	0.005026508	0.007571305	0.005028392
0.03	0.009841498	0.007547977	0.01023166	0.007551797	0.010623342	0.007555771	0.011016622	0.007559901	0.01141158	0.007564188
0.04	0.013163382	0.01008556	0.013690578	0.010092409	0.014220533	0.01009954	0.014753397	0.010106957	0.015289323	0.010114663
0.05	0.016506419	0.01263411	0.017174295	0.012644902	0.017846576	0.012656149	0.018523512	0.012667857	0.019205358	0.012680033
0.06	0.019870896	0.015193732	0.020683198	0.015209404	0.021501981	0.015225752	0.022327628	0.015242787	0.023160539	0.015260521
0.07	0.023257103	0.017764532	0.024217679	0.017786046	0.025187269	0.017808508	0.026166431	0.017831937	0.027155749	0.017856352
0.08	0.026665339	0.020346619	0.027778142	0.020374961	0.028902979	0.02040458	0.030040628	0.020435503	0.031191904	0.020467762
0.09	0.030095909	0.022940104	0.031365002	0.022976285	0.032649665	0.023014132	0.033950947	0.023053685	0.035269955	0.023094991
0.1	0.033549123	0.025545099	0.034978683	0.025590156	0.036427897	0.025637333	0.037898142	0.025686688	0.039390888	0.025738285
0.11	0.0370253	0.028161717	0.038619621	0.028216715	0.04023826	0.028274356	0.041882994	0.028334721	0.043555726	0.028397898
0.12	0.040524765	0.030790074	0.042288263	0.030856105	0.04408136	0.030925377	0.045906308	0.030997999	0.047765532	0.031074091
0.13	0.044047851	0.03343029	0.045985066	0.033508473	0.047957818	0.033590576	0.049968917	0.033676744	0.05202141	0.033767133
0.14	0.047594897	0.036082484	0.049710503	0.036173968	0.051868276	0.036270139	0.054071686	0.036371182	0.05632451	0.036477299
0.15	0.051166252	0.038746777	0.053465056	0.038852743	0.055813396	0.038964254	0.058215506	0.039081545	0.060676026	0.039204875
0.16	0.054762272	0.041423295	0.057249221	0.041544954	0.05979386	0.041673114	0.062401305	0.041808072	0.065077202	0.041950154
0.17	0.058383321	0.044112163	0.06106351	0.044250758	0.063810371	0.044396917	0.066630041	0.044551007	0.069529335	0.044713437
0.18	0.062029772	0.046813511	0.064908445	0.046970318	0.067863655	0.047135864	0.07090271	0.047310602	0.074033776	0.047495036
0.19	0.065702007	0.049527469	0.068784566	0.0497038	0.071954462	0.049890165	0.075220344	0.050087116	0.078591933	0.050295272
0.2	0.069400417	0.052254171	0.072692427	0.052451371	0.076083567	0.052660029	0.079584015	0.052880815	0.083205278	0.053114476
0.21	0.073125403	0.054993753	0.076632597	0.055213204	0.08025177	0.055445676	0.083994834	0.055691971	0.087875347	0.055952991
0.22	0.076877374	0.057746352	0.080605663	0.057989476	0.084459899	0.058247328	0.08845396	0.058520866	0.092603747	0.05881117
0.23	0.080656751	0.060512111	0.084612228	0.060780366	0.088708808	0.061065214	0.092962594	0.061367789	0.097392157	0.061689378
0.24	0.084463965	0.063291172	0.088652913	0.063586058	0.092999383	0.063899568	0.097521988	0.064233037	0.102242336	0.064587993
0.25	0.088299457	0.066083681	0.092728357	0.06640674	0.09733254	0.066750629	0.102133444	0.067116918	0.107156128	0.067507406
0.26	0.092163681	0.068889788	0.09683922	0.069242604	0.101709227	0.069618646	0.106798319	0.070019747	0.112135465	0.07044802
0.27	0.096057101	0.071709644	0.10098618	0.072093846	0.106130428	0.072503871	0.111518027	0.07294185	0.117182377	0.073410255
0.28	0.099980194	0.074543405	0.105169937	0.074960668	0.110597162	0.075406564	0.116294043	0.075883563	0.122298995	0.076394546
0.29	0.103933448	0.077391228	0.109391213	0.077843274	0.115110484	0.078326992	0.121127907	0.078845233	0.12748756	0.079401342
0.3	0.107917365	0.080253274	0.113650753	0.080741876	0.119671492	0.081265429	0.126021228	0.081827217	0.132750431	0.08243111
0.31	0.11193246	0.083129707	0.117949325	0.083656689	0.124281323	0.084222158	0.130975685	0.084829884	0.138090095	0.085484337
0.32	0.115979264	0.086020694	0.122287722	0.086587932	0.12894116	0.087197468	0.135993038	0.087853617	0.143509173	0.088561525
0.33	0.120058318	0.088926407	0.126666764	0.089535832	0.13365223	0.090191657	0.141075127	0.09089881	0.149010436	0.0916632
0.34	0.124170183	0.09184702	0.131087296	0.09250062	0.138415811	0.093205033	0.146223882	0.093965871	0.15459681	0.094789908
0.35	0.128315431	0.09478271	0.135550195	0.095482533	0.143233234	0.096237911	0.151441324	0.097055223	0.160271395	0.097942217
0.36	0.132494652	0.09773366	0.140056364	0.098481814	0.14810588	0.099290617	0.156729578	0.100167303	0.166037476	0.10112072

	ψ=0.6		ψ=0.7		ψ=0.8		ψ=0.9		ψ=1	
Z	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**
0.37	0.136708455	0.100700055	0.144606739	0.101498712	0.153035193	0.102363486	0.162090874	0.103302564	0.171898538	0.104326036
0.38	0.140957462	0.103682084	0.14920229	0.104533482	0.158022674	0.105456862	0.167527556	0.106461478	0.177858286	0.10755881
0.39	0.145242317	0.106679941	0.153844019	0.107586387	0.163069893	0.108571103	0.173042093	0.109644531	0.183920664	0.110819718
0.4	0.14956368	0.109693823	0.158532965	0.110657695	0.168178484	0.111706576	0.178637087	0.11285223	0.190089873	0.114109467
0.41	0.153922233	0.112723932	0.163270205	0.113747682	0.173350158	0.114863661	0.18431528	0.116085102	0.196370403	0.117428798
0.42	0.158318676	0.115770476	0.168056856	0.116856633	0.178586702	0.118042748	0.190079571	0.119343692	0.202767052	0.120778487
0.43	0.162753733	0.118833664	0.172894075	0.119984837	0.183889985	0.121244243	0.195933021	0.122628572	0.209284962	0.124159349
0.44	0.167228147	0.121913714	0.177783066	0.123132595	0.189261965	0.124468565	0.201878873	0.125940334	0.215929653	0.127572243
0.45	0.171742687	0.125010845	0.182725078	0.126300214	0.194704694	0.127716145	0.207920563	0.129279595	0.222707062	0.13101807
0.46	0.176298145	0.128125284	0.187721407	0.129488011	0.200220323	0.130987432	0.214061737	0.132647001	0.229623588	0.134497781
0.47	0.180895337	0.131257262	0.192773403	0.13269631	0.205811111	0.13428289	0.220306269	0.136043225	0.236686143	0.138012382
0.48	0.185535108	0.134407016	0.19788247	0.135925448	0.21147943	0.137602999	0.226658286	0.139468971	0.243902212	0.141562935
0.49	0.190218328	0.137574788	0.203050069	0.139175768	0.217227777	0.140948258	0.233122181	0.142924974	0.251279921	0.145150563
0.5	0.194945897	0.140760826	0.208277722	0.142447626	0.223058778	0.144319184	0.239702649	0.146412007	0.258828115	0.148776461
0.51	0.199718745	0.143965385	0.213567015	0.145741389	0.228975204	0.147716312	0.246404709	0.149930878	0.266556449	0.152441897
0.52	0.204537834	0.147188725	0.218919603	0.149057433	0.234979974	0.151140202	0.253233738	0.153482435	0.274475497	0.156148223
0.53	0.209404156	0.150431113	0.224337214	0.152396149	0.241076174	0.154591432	0.260195508	0.157067571	0.282596875	0.15989688
0.54	0.214318742	0.153692823	0.229821649	0.155757937	0.247267066	0.158070607	0.267296228	0.160687225	0.290933392	0.163689411
0.55	0.219282656	0.156974137	0.235374796	0.159143213	0.253556104	0.161578354	0.274542588	0.164342386	0.299499227	0.167527471
0.56	0.224297002	0.160275341	0.240998626	0.162552404	0.25994695	0.165115328	0.281941816	0.168034099	0.308310138	0.171412839
0.57	0.229362922	0.163596733	0.246695204	0.165985955	0.26644349	0.168682213	0.289501738	0.171763468	0.317383721	0.175347436
0.58	0.234481601	0.166938616	0.252466692	0.169444321	0.273049856	0.172279721	0.297230851	0.175531661	0.326739724	0.179333337
0.59	0.239654269	0.170301302	0.258315359	0.172927979	0.279770444	0.175908599	0.3051384	0.17933992	0.336400423	0.183372797
0.6	0.2448822	0.173685111	0.264243586	0.176437416	0.286609946	0.179569626	0.313234482	0.18318956	0.346391106	0.187468275
0.61	0.250166721	0.177090372	0.270253872	0.179973143	0.293573368	0.18326362	0.32153015	0.187081985	0.356740665	0.191622459
0.62	0.255509205	0.180517424	0.276348849	0.183535684	0.300666068	0.186991436	0.330037548	0.191018692	0.367482352	0.195838306
0.63	0.260911084	0.183966615	0.282531283	0.187125587	0.307893787	0.190753974	0.338770067	0.195001282	0.378654755	0.200119087
0.64	0.266373845	0.187438304	0.28880409	0.190743418	0.315262692	0.19455218	0.347742528	0.199031471	0.390303073	0.204468436
0.65	0.271899036	0.190932859	0.295170347	0.194389766	0.322779415	0.198387048	0.356971404	0.203111104	0.402480799	0.208890419
0.66	0.27748827	0.19445066	0.301633301	0.198065242	0.330451108	0.202259626	0.36647509	0.207242172	0.415252012	0.213389625
0.67	0.283143225	0.197992099	0.308196386	0.201770484	0.338285502	0.206171021	0.376274228	0.211426825	0.428694539	0.217971269
0.68	0.288865654	0.201557577	0.314863237	0.205506154	0.346290969	0.210122402	0.386392106	0.2156674	0.442904454	0.222641345
0.69	0.294657385	0.205147512	0.321637703	0.209272942	0.354476605	0.214115008	0.396855157	0.219966441	0.458002642	0.227406824
0.7	0.300520325	0.208762331	0.328523874	0.213071569	0.362852313	0.218150153	0.407693584	0.224326732	0.474144771	0.232275923
0.71	0.306456468	0.212402478	0.335526093	0.216902785	0.371428912	0.222229232	0.418942148	0.228751333	0.491537055	0.23725851
0.72	0.312467901	0.216068409	0.342648983	0.220767376	0.380218255	0.226353733	0.430641193	0.233243625	0.510462594	0.242366685

	ψ=0.6		ψ=0.7		ψ=0.8		ψ=0.9		ψ=1	
Z	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**
0.73	0.318556805	0.219760596	0.349897471	0.224666163	0.389233375	0.230525242	0.442837971	0.237807364	0.531328453	0.247615719
0.74	0.324725465	0.223479528	0.357276818	0.228600006	0.39848865	0.234745457	0.455588415	0.242446755	0.554757834	0.253025622
0.75	0.330976277	0.227225709	0.36479265	0.232569805	0.408000011	0.239016197	0.468959531	0.247166534	0.581794822	0.258624064
0.76	0.337311756	0.230999661	0.372450995	0.236576506	0.417785177	0.243339419	0.483032714	0.251972087	0.614455993	0.264452635
0.77	0.34373454	0.234801926	0.380258323	0.240621102	0.427863954	0.247717234	0.497908456	0.256869599	0.657828928	0.270583972
0.78	0.350247403	0.238633063	0.388221592	0.244704637	0.438258593	0.252151926	0.513713234	0.26186626	1	0.28
0.79	0.356853266	0.242493653	0.396348304	0.248828213	0.448994223	0.256645973	0.530609963	0.266970543	1	0.29
0.82	0.377260437	0.254258288	0.421793551	0.261451123	0.483554173	0.270512593	0.590463697	0.283043255	1	0.32
0.83	0.384270773	0.258242955	0.430662179	0.265747154	0.495985238	0.275273851	0.614989046	0.288708148	1	0.33
0.84	0.391391283	0.262260334	0.439742336	0.270089748	0.508951448	0.280110977	0.643297214	0.294568115	1	0.34
0.85	0.398626018	0.266311155	0.449046417	0.27448046	0.522513814	0.285028451	0.67751445	0.300666004	1	0.35
0.86	0.405979275	0.270396182	0.458588043	0.27892095	0.53674563	0.29003133	0.723039925	0.307076815	1	0.36
0.87	0.413455619	0.274516211	0.468382232	0.283412989	0.551736217	0.295125369	1	0.314776275	1	0.37
0.88	0.421059902	0.278672074	0.478445611	0.287958478	0.567596277	0.300317191	1	0.324141519	1	0.38
0.89	0.428797296	0.282864639	0.488796661	0.292559458	0.584465785	0.305614512	1	0.333506764	1	0.39
0.9	0.436673318	0.287094815	0.499456026	0.297218128	0.602526057	0.311026452	1	0.342872008	1	0.4
0.91	0.444693865	0.291363555	0.510446873	0.301936866	0.622019087	0.316563992	1	0.352237253	1	0.41
0.92	0.45286525	0.295671856	0.52179535	0.306718252	0.643280292	0.322240662	1	0.361602497	1	0.42
0.93	0.461194243	0.300020768	0.533531142	0.311565093	0.666798201	0.328073638	1	0.370967742	1	0.43
0.94	0.469688122	0.304411391	0.54568818	0.316480459	0.693334344	0.334085635	1	0.380332986	1	0.44
0.95	0.478354724	0.308844885	0.558305536	0.321467724	0.724199899	0.340308511	1	0.389698231	1	0.45
0.96	0.487202508	0.313322472	0.571428571	0.326530612	0.762048839	0.346791346	1	0.399063476	1	0.46
0.97	0.496240626	0.317845444	0.585110452	0.33167326	0.814321517	0.35362461	1	0.40842872	1	0.47
0.98	0.505479002	0.322415163	0.599414151	0.336900293	1	0.361328125	1	0.417793965	1	0.48
0.99	0.51492843	0.327033076	0.614415166	0.342216921	1	0.370117188	1	0.427159209	1	0.49
1	0.52460068	0.331700715	0.630205298	0.347629065	1	0.37890625	1	0.436524454	1	0.5
1.01	0.534508626	0.336419709	0.646898016	0.353143521	1	0.387695313	1	0.445889698	1	0.51
1.02	0.544666395	0.341191796	0.664636338	0.358768186	1	0.396484375	1	0.455254943	1	0.52
1.03	0.555089547	0.346018829	0.683604806	0.364512367	1	0.405273438	1	0.464620187	1	0.53
1.04	0.56579528	0.350902792	0.704048545	0.370387229	1	0.4140625	1	0.473985432	1	0.54
1.05	0.576802685	0.355845813	0.726305246	0.376406469	1	0.422851563	1	0.483350676	1	0.55
1.06	0.588133045	0.360850184	0.75086279	0.382587377	1	0.431640625	1	0.492715921	1	0.56
1.07	0.599810204	0.365918376	0.778473164	0.388952642	1	0.440429688	1	0.502081165	1	0.57
1.08	0.611861018	0.371053067	0.810409287	0.39553374	1	0.44921875	1	0.51144641	1	0.58
1.09	0.62431591	0.376257166	0.849173628	0.402378309	1	0.458007813	1	0.520811655	1	0.59
1.1	0.637209567	0.381533849	0.901330424	0.409570991	1	0.466796875	1	0.530176899	1	0.6

	ψ=0.6		ψ=0.7		ψ=0.8		ψ=0.9		ψ=1	
Z	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**
1.11	0.650581823	0.386886597	1	0.417355372	1	0.475585938	1	0.539542144	1	0.61
1.12	0.664478788	0.392319245	1	0.425619835	1	0.484375	1	0.548907388	1	0.62
1.13	0.678954315	0.39783604	1	0.433884298	1	0.493164063	1	0.558272633	1	0.63
1.14	0.694071946	0.403441719	1	0.44214876	1	0.501953125	1	0.567637877	1	0.64
1.15	0.709907524	0.409141599	1	0.450413223	1	0.510742188	1	0.577003122	1	0.65
1.16	0.726552793	0.4149417	1	0.458677686	1	0.51953125	1	0.586368366	1	0.66
1.17	0.744120474	0.420848903	1	0.466942149	1	0.528320313	1	0.595733611	1	0.67
1.18	0.76275162	0.426871164	1	0.475206612	1	0.537109375	1	0.605098855	1	0.68
1.19	0.782626675	0.433017803	1	0.483471074	1	0.545898438	1	0.6144641	1	0.69
1.22	0.852562019	0.452327923	1	0.508264463	1	0.572265625	1	0.642559834	1	0.72
1.23	0.880930137	0.459112405	1	0.516528926	1	0.581054688	1	0.651925078	1	0.73
1.24	0.913368676	0.466113972	1	0.524793388	1	0.58984375	1	0.661290323	1	0.74
1.25	0.951954325	0.473375709	1	0.533057851	1	0.598632813	1	0.670655567	1	0.75
1.26	1	0.480968858	1	0.541322314	1	0.607421875	1	0.680020812	1	0.76
1.27	1	0.488754325	1	0.549586777	1	0.616210938	1	0.689386056	1	0.77
1.28	1	0.496539792	1	0.55785124	1	0.625	1	0.698751301	1	0.78
1.29	1	0.50432526	1	0.566115702	1	0.633789063	1	0.708116545	1	0.79
1.3	1	0.512110727	1	0.574380165	1	0.642578125	1	0.71748179	1	0.8
1.31	1	0.519896194	1	0.582644628	1	0.651367188	1	0.726847034	1	0.81
1.32	1	0.527681661	1	0.590909091	1	0.66015625	1	0.736212279	1	0.82
1.33	1	0.535467128	1	0.599173554	1	0.668945313	1	0.745577523	1	0.83
1.34	1	0.543252595	1	0.607438017	1	0.677734375	1	0.754942768	1	0.84
1.35	1	0.551038062	1	0.615702479	1	0.686523438	1	0.764308012	1	0.85
1.36	1	0.558823529	1	0.623966942	1	0.6953125	1	0.773673257	1	0.86
1.37	1	0.566608997	1	0.632231405	1	0.704101563	1	0.783038502	1	0.87
1.38	1	0.574394464	1	0.640495868	1	0.712890625	1	0.792403746	1	0.88
1.39	1	0.582179931	1	0.648760331	1	0.721679688	1	0.801768991	1	0.89
1.4	1	0.589965398	1	0.657024793	1	0.73046875	1	0.811134235	1	0.9
1.41	1	0.597750865	1	0.665289256	1	0.739257813	1	0.82049948	1	0.91
1.42	1	0.605536332	1	0.673553719	1	0.748046875	1	0.829864724	1	0.92
1.43	1	0.613321799	1	0.681818182	1	0.756835938	1	0.839229969	1	0.93
1.44	1	0.621107266	1	0.690082645	1	0.765625	1	0.848595213	1	0.94
1.45	1	0.628892734	1	0.698347107	1	0.774414063	1	0.857960458	1	0.95
1.46	1	0.636678201	1	0.70661157	1	0.783203125	1	0.867325702	1	0.96
1.47	1	0.644463668	1	0.714876033	1	0.791992188	1	0.876690947	1	0.97
1.48	1	0.652249135	1	0.723140496	1	0.80078125	1	0.886056191	1	0.98

	ψ=0.6		ψ=0.7		ψ=0.8		ψ=0.9		ψ=1	
Z	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**
1.49	1	0.660034602	1	0.731404959	1	0.809570313	1	0.895421436	1	0.99
1.5	1	0.667820069	1	0.739669421	1	0.818359375	1	0.904786681	1	1
1.51	1	0.675605536	1	0.747933884	1	0.827148438	1	0.914151925	1	1.01
1.52	1	0.683391003	1	0.756198347	1	0.8359375	1	0.92351717	1	1.02
1.53	1	0.691176471	1	0.76446281	1	0.844726563	1	0.932882414	1	1.03
1.54	1	0.698961938	1	0.772727273	1	0.853515625	1	0.942247659	1	1.04
1.55	1	0.706747405	1	0.780991736	1	0.862304688	1	0.951612903	1	1.05
1.56	1	0.714532872	1	0.789256198	1	0.87109375	1	0.960978148	1	1.06
1.57	1	0.722318339	1	0.797520661	1	0.879882813	1	0.970343392	1	1.07
1.58	1	0.730103806	1	0.805785124	1	0.888671875	1	0.979708637	1	1.08
1.59	1	0.737889273	1	0.814049587	1	0.897460938	1	0.989073881	1	1.09
1.6	1	0.74567474	1	0.82231405	1	0.90625	1	0.998439126	1	1.1
1.61	1	0.753460208	1	0.830578512	1	0.915039063	1	1.00780437	1	1.11
1.62	1	0.761245675	1	0.838842975	1	0.923828125	1	1.017169615	1	1.12
1.63	1	0.769031142	1	0.847107438	1	0.932617188	1	1.02653486	1	1.13
1.64	1	0.776816609	1	0.855371901	1	0.94140625	1	1.035900104	1	1.14
1.65	1	0.784602076	1	0.863636364	1	0.950195313	1	1.045265349	1	1.15
1.66	1	0.792387543	1	0.871900826	1	0.958984375	1	1.054630593	1	1.16
1.67	1	0.80017301	1	0.880165289	1	0.967773438	1	1.063995838	1	1.17
1.68	1	0.807958478	1	0.888429752	1	0.9765625	1	1.073361082	1	1.18
1.69	1	0.815743945	1	0.896694215	1	0.985351563	1	1.082726327	1	1.19
1.7	1	0.823529412	1	0.904958678	1	0.994140625	1	1.092091571	1	1.2
1.71	1	0.831314879	1	0.91322314	1	1.002929688	1	1.101456816	1	1.21
1.72	1	0.839100346	1	0.921487603	1	1.01171875	1	1.11082206	1	1.22
1.73	1	0.846885813	1	0.929752066	1	1.020507813	1	1.120187305	1	1.23
1.74	1	0.85467128	1	0.938016529	1	1.029296875	1	1.129552549	1	1.24
1.75	1	0.862456747	1	0.946280992	1	1.038085938	1	1.138917794	1	1.25
1.76	1	0.870242215	1	0.954545455	1	1.046875	1	1.148283039	1	1.26
1.77	1	0.878027682	1	0.962809917	1	1.055664063	1	1.157648283	1	1.27
1.78	1	0.885813149	1	0.97107438	1	1.064453125	1	1.167013528	1	1.28
1.79	1	0.893598616	1	0.979338843	1	1.073242188	1	1.176378772	1	1.29
1.8	1	0.901384083	1	0.987603306	1	1.08203125	1	1.185744017	1	1.3
1.81	1	0.90916955	1	0.995867769	1	1.090820313	1	1.195109261	1	1.31
1.82	1	0.916955017	1	1.004132231	1	1.099609375	1	1.204474506	1	1.32
1.83	1	0.924740484	1	1.012396694	1	1.108398438	1	1.21383975	1	1.33
1.84	1	0.932525952	1	1.020661157	1	1.1171875	1	1.223204995	1	1.34

	ψ=0.6		ψ=0.7		ψ=0.8		ψ=0.9		ψ=1	
Z	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**
1.85	1	0.940311419	1	1.02892562	1	1.125976563	1	1.232570239	1	1.35
1.86	1	0.948096886	1	1.037190083	1	1.134765625	1	1.241935484	1	1.36
1.87	1	0.955882353	1	1.045454545	1	1.143554688	1	1.251300728	1	1.37
1.88	1	0.96366782	1	1.053719008	1	1.15234375	1	1.260665973	1	1.38
1.89	1	0.971453287	1	1.061983471	1	1.161132813	1	1.270031217	1	1.39
1.9	1	0.979238754	1	1.070247934	1	1.169921875	1	1.279396462	1	1.4
1.91	1	0.987024221	1	1.078512397	1	1.178710938	1	1.288761707	1	1.41
1.92	1	0.994809689	1	1.08677686	1	1.1875	1	1.298126951	1	1.42
1.93	1	1.002595156	1	1.095041322	1	1.196289063	1	1.307492196	1	1.43
1.94	1	1.010380623	1	1.103305785	1	1.205078125	1	1.31685744	1	1.44
1.95	1	1.01816609	1	1.111570248	1	1.213867188	1	1.326222685	1	1.45
1.96	1	1.025951557	1	1.119834711	1	1.22265625	1	1.335587929	1	1.46
1.97	1	1.033737024	1	1.128099174	1	1.231445313	1	1.344953174	1	1.47
1.98	1	1.041522491	1	1.136363636	1	1.240234375	1	1.354318418	1	1.48
1.99	1	1.049307958	1	1.144628099	1	1.249023438	1	1.363683663	1	1.49
2	1	1.057093426	1	1.152892562	1	1.2578125	1	1.373048907	1	1.5
2.01	1	1.064878893	1	1.161157025	1	1.266601563	1	1.382414152	1	1.51
2.02	1	1.07266436	1	1.169421488	1	1.275390625	1	1.391779396	1	1.52
2.03	1	1.080449827	1	1.17768595	1	1.284179688	1	1.401144641	1	1.53
2.04	1	1.088235294	1	1.185950413	1	1.29296875	1	1.410509886	1	1.54
2.05	1	1.096020761	1	1.194214876	1	1.301757813	1	1.41987513	1	1.55
2.06	1	1.103806228	1	1.202479339	1	1.310546875	1	1.429240375	1	1.56
2.07	1	1.111591696	1	1.210743802	1	1.319335938	1	1.438605619	1	1.57
2.08	1	1.119377163	1	1.219008264	1	1.328125	1	1.447970864	1	1.58
2.09	1	1.12716263	1	1.227272727	1	1.336914063	1	1.457336108	1	1.59
2.1	1	1.134948097	1	1.23553719	1	1.345703125	1	1.466701353	1	1.6
2.11	1	1.142733564	1	1.243801653	1	1.354492188	1	1.476066597	1	1.61
2.12	1	1.150519031	1	1.252066116	1	1.36328125	1	1.485431842	1	1.62
2.13	1	1.158304498	1	1.260330579	1	1.372070313	1	1.494797086	1	1.63
2.14	1	1.166089965	1	1.268595041	1	1.380859375	1	1.504162331	1	1.64
2.15	1	1.173875433	1	1.276859504	1	1.389648438	1	1.513527575	1	1.65
2.16	1	1.1816609	1	1.285123967	1	1.3984375	1	1.52289282	1	1.66
2.17	1	1.189446367	1	1.29338843	1	1.407226563	1	1.532258065	1	1.67
2.18	1	1.197231834	1	1.301652893	1	1.416015625	1	1.541623309	1	1.68
2.19	1	1.205017301	1	1.309917355	1	1.424804688	1	1.550988554	1	1.69

	ψ=0.6		ψ=0.7		ψ=0.8		ψ=0.9		ψ=1	
Z	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**	**Optimal γ**	**Optimized Profits**
2.2	1	1.212802768	1	1.318181818	1	1.43359375	1	1.560353798	1	1.7
2.21	1	1.220588235	1	1.326446281	1	1.442382813	1	1.569719043	1	1.71
2.22	1	1.228373702	1	1.334710744	1	1.451171875	1	1.579084287	1	1.72
2.23	1	1.23615917	1	1.342975207	1	1.459960938	1	1.588449532	1	1.73
2.24	1	1.243944637	1	1.351239669	1	1.46875	1	1.597814776	1	1.74
2.25	1	1.251730104	1	1.359504132	1	1.477539063	1	1.607180021	1	1.75
2.26	1	1.259515571	1	1.367768595	1	1.486328125	1	1.616545265	1	1.76
2.27	1	1.267301038	1	1.376033058	1	1.495117188	1	1.62591051	1	1.77
2.28	1	1.275086505	1	1.384297521	1	1.50390625	1	1.635275754	1	1.78
2.29	1	1.282871972	1	1.392561983	1	1.512695313	1	1.644640999	1	1.79
2.3	1	1.290657439	1	1.400826446	1	1.521484375	1	1.654006243	1	1.8
2.31	1	1.298442907	1	1.409090909	1	1.530273438	1	1.663371488	1	1.81
2.32	1	1.306228374	1	1.417355372	1	1.5390625	1	1.672736733	1	1.82
2.33	1	1.314013841	1	1.425619835	1	1.547851563	1	1.682101977	1	1.83
2.34	1	1.321799308	1	1.433884298	1	1.556640625	1	1.691467222	1	1.84
2.35	1	1.329584775	1	1.44214876	1	1.565429688	1	1.700832466	1	1.85
2.36	1	1.337370242	1	1.450413223	1	1.57421875	1	1.710197711	1	1.86
2.37	1	1.345155709	1	1.458677686	1	1.583007813	1	1.719562955	1	1.87
2.38	1	1.352941176	1	1.466942149	1	1.591796875	1	1.7289282	1	1.88
2.39	1	1.360726644	1	1.475206612	1	1.600585938	1	1.738293444	1	1.89
2.4	1	1.368512111	1	1.483471074	1	1.609375	1	1.747658689	1	1.9
2.41	1	1.376297578	1	1.491735537	1	1.618164063	1	1.757023933	1	1.91
2.42	1	1.384083045	1	1.5	1	1.626953125	1	1.766389178	1	1.92
2.43	1	1.391868512	1	1.508264463	1	1.635742188	1	1.775754422	1	1.93
2.44	1	1.399653979	1	1.516528926	1	1.64453125	1	1.785119667	1	1.94
2.45	1	1.407439446	1	1.524793388	1	1.653320313	1	1.794484912	1	1.95
2.46	1	1.415224913	1	1.533057851	1	1.662109375	1	1.803850156	1	1.96
2.47	1	1.423010381	1	1.541322314	1	1.670898438	1	1.813215401	1	1.97
2.48	1	1.430795848	1	1.549586777	1	1.6796875	1	1.822580645	1	1.98
2.49	1	1.438581315	1	1.55785124	1	1.688476563	1	1.83194589	1	1.99
2.5	1	1.446366782	1	1.566115702	1	1.697265625	1	1.841311134	1	2

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